Dynamic Observers for the Synthesis of Opaque Systems

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Context

Need for Security in Transactional Systems

- Web-services: e-banking, online transactions
- Id documents: biometric passport, Medicare Card
- E-voting systems

Different Types of Security

- Integrity: illegal actions cannot be performed by an unauthorized user
  Bank account management cannot be managed by a third party
- Availability: some actions must be available
  Withdrawing money from your bank account
- Privacy: information should remain hidden from some users
  PIN code

Opacity was introduced in [Mazaré, 2004, Bryans et al., 2008]

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In this paper we consider opacity (privacy)

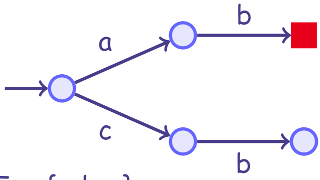
Opacity was introduced in [Mazaré, 2004, Bryans et al., 2008]

Outline of the Talk

Outline

What is Opacity?

System S



$\Sigma = \{a, b, c\}$

Secret F

Secret = set of states ■

Events in $\Sigma_o \subseteq \Sigma$ are observable

Example: $\Sigma_o = \{b\}$

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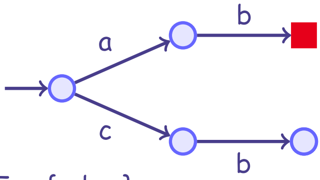

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Opacity: an external observer should never know F-states

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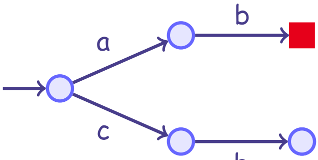

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Secret F is opaque

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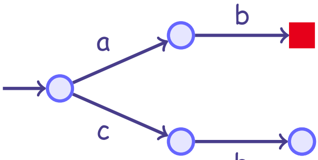

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Secret F is not opaque

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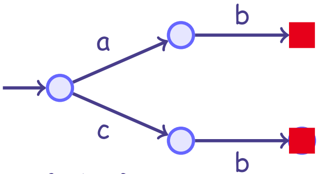

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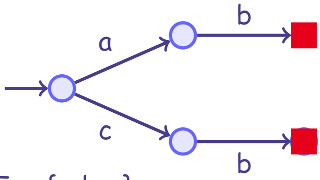

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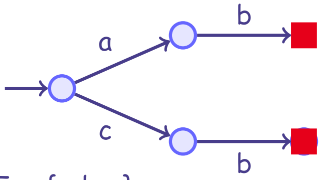

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To check opacity: use your favorite Formal Method:

- Model-checking
- Theorem proving
- Tools to support automatic analysis of systems

Opacity for Non-Deterministic Automata

$A = (Q, q_0, \Sigma, \delta, F)$ a NDA

$\Sigma_o \subseteq \Sigma$ set of observable events

F set of secret states

NDA A $\xrightarrow{u \in \Sigma^*}$ Projection P $\xrightarrow{v = P(u) \in \Sigma_o^*}$ Attacker U

Assumptions

- Attacker knows A and the projection P/alphabet Σ_o
- $K_{\Sigma_o}(v)$: knowledge set (of states) of the attacker after observing v

Definition (Opacity)

F is opaque w.r.t. (A, Σ_o) if $\forall v \in P(Tr(A)), K_{\Sigma_o}(v) \not\subseteq F$ $(K_{\Sigma_o}(v) \cap (Q \setminus F) \neq \emptyset)$.

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Input: A NDA A, F secret set of states, Σ_o set of observable events.
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$$\begin{tikzcd} \boxed{\text{NDA } A} \arrow[r, "u \in \Sigma^*"] & \boxed{\text{Projection } P} \arrow[r, "v = P(u) \in \Sigma_o^*"] & \boxed{\text{Attacker } U} \end{tikzcd}$$

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Knowledge Set of the Attacker

- $Tr(A)$ = set of words generated by A
- P is the projection over $\Sigma_o \subseteq \Sigma$
- $P^{-1}(w)$ = set of words which project onto w $\qquad P^{-1} : \Sigma_o^* \longrightarrow 2^{\Sigma^*}$
- $Pre(\varepsilon) = \{\varepsilon\}$ and $Pre(u.\lambda) = P^{-1}(u).\lambda \cap Tr(A)$
- Knowledge set of U: $K_{\Sigma_o}(u) = \delta(q_0, Pre(u))$

Consider knowledge set right after each observation of the attacker

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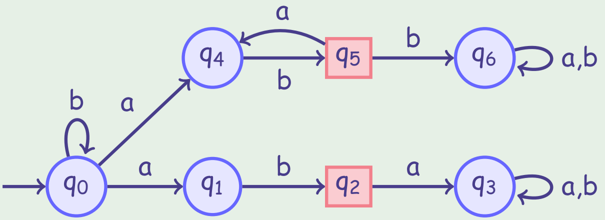
Example

$\Sigma_o = \{b\}$
$P(b.b.a.b.a) = b.b.b$
$P^{-1}(b) = a^*.b.a^*$
$Pre(b) = \{b, a.b\}$
$K_{\Sigma_o}(b) = \{q_0, q_5, q_2\}$



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Problem 1: Checking opacity with Static Filters

Input: a NDA A, F secret set of states, Σ_o set of observable events.
Problem: Is F opaque w.r.t. (A, Σ_o) ?

Theorem

Problem 1 is PSPACE-complete.

Algorithms for Checking Opacity

Proof.

Reduction of universality problem for non-deterministic automaton.
Given A over Σ with accepting states F, the universality problem is:

$$\text{decide whether } L_F(A) = \Sigma^*.$$

Assume A is complete i.e. $Tr(A) = \Sigma^*$.
Reduction:

$$A \text{ is universal iff } Q \setminus F \text{ is opaque for } (A, \Sigma).$$

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Algorithm to Check Opacity

1. Subset construction
2. check whether a subset $S \subseteq F$ is reachable

What if the system is NOT opaque ?

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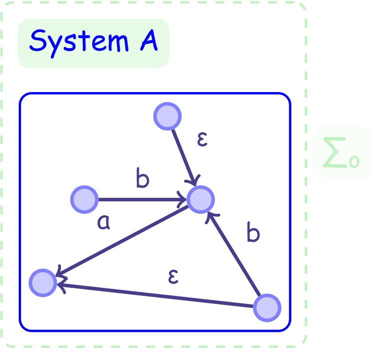
Enforcing Opacity



- Either restrict set of behaviours: add a controller C

  [Dubreil et al., 2008]

- Or modify the set of observable events to $\Sigma_o' \neq \Sigma_o$

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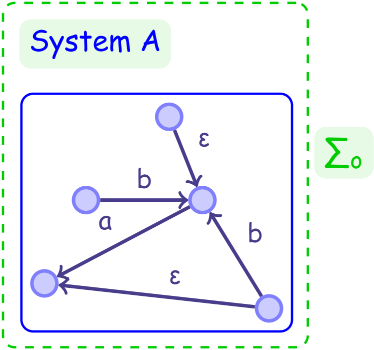

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(A, Σ_o) is NOT opaque

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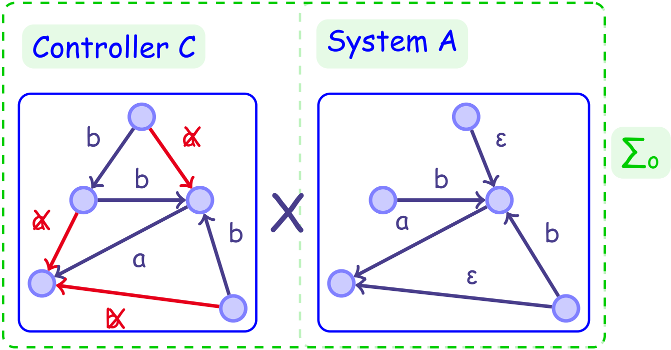

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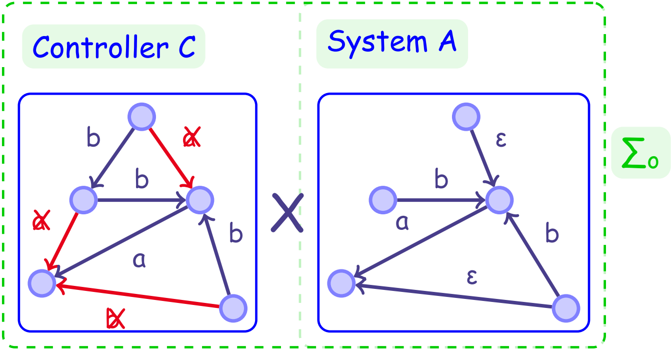

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Ensure $(C \times A, \Sigma_o)$ is opaque

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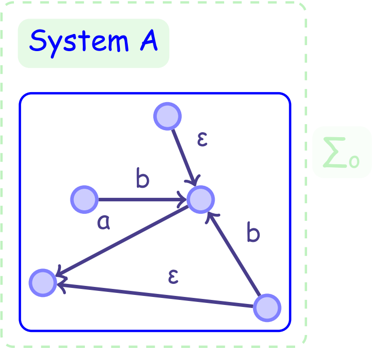

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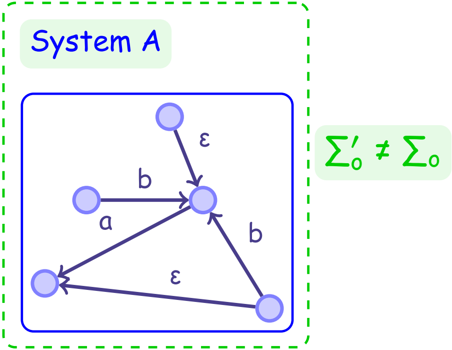

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Outline

Minimization of the Set of Hidden Events

- Events = services provided to (external) users
- Hiding events = restricting services

Goal: ensure opacity while preserving services

Problem 2: Static Minimization Problem

Input: $A = (Q, q_0, \Sigma, \delta, F)$ a NDA, F secret set of states and $n \in \mathbb{N}$.
Problem: Is there any $\Sigma_o \subseteq \Sigma$ with $|\Sigma_o| \geq n$ s.t. F is opaque w.r.t. (A, Σ_o) ?

Theorem

Problem 2 is PSPACE-complete.

Computing the maximum n is also PSPACE-complete.

Can we do any better ?

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Using Dynamic Filters

- **Static Filter:** $\Sigma_o = \{a\}$ or $\Sigma_o = \{b\} \Rightarrow F$ is opaque
  Must hide at least one event
- **Dynamic Filter:** Hide b after the observation of an a and let everything be observable after the observation of a second a

Result: Events are more often visible

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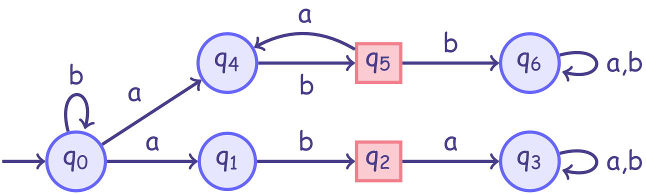

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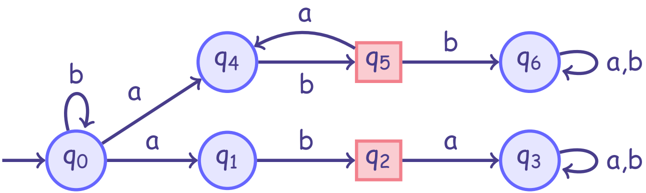

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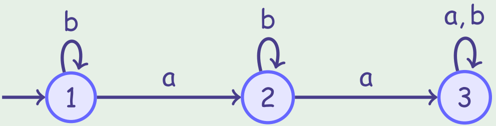
Dynamic Filters

Definition (Dynamic Filter)

A dynamic filter is a complete (infinite) deterministic labeled transition system $\Phi = (X, x_0, \Sigma, \delta_o, L)$ where

- $L : X \longrightarrow 2^{\Sigma}$ is a labeling function that specifies the set of events that are observable at state x;
- For all $x \in X$ and for all $\lambda \in \Sigma$, if $\lambda \notin L(x)$, then $\delta_o(x, \lambda) = x$.

Example (Finite State Filter)



$L(1) = \{a, b\}$ $\quad$ $L(2) = \{a\}$ $\quad$ $L(3) = \{a, b\}$

Φ is also a transducer

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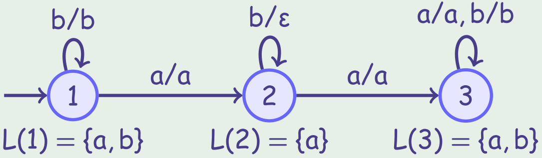

Φ is also a **transducer**

Opacity with Dynamic Filters

- $\Phi^{-1}(w)$ = set of words u that filter onto w (s.t. $\Phi(u) = w$)
- $Pre(\varepsilon) = \{\varepsilon\}$ and $Pre(u.\lambda) = \Phi^{-1}(u).\lambda \cap Tr(A)$
- Knowledge set of attacker: $K_\Phi(u) = \delta(q_0, Pre(u))$

Definition (Opacity)

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Problem 3: Opacity Problem with Dynamic Filters

Input: A a NDA, F secret set of states, Φ a filter.
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Issues

- How to check opacity with a dynamic filter?
- How to compare dynamic filters?
- How to synthesize optimal dynamic filters?

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Checking Opacity for Finite State Filters

Opacity for Finite State Filters

Input: A, F, Φ a finite state filter.
Problem: Is F opaque w.r.t. (A, Φ) ?

To check opacity, build a product $A \otimes \Phi$

1. initial state (q_0, x_0)
2. $(q, x) \xrightarrow{\lambda} (q', x')$ iff $q \xrightarrow{\lambda}_A q'$, $x \xrightarrow{\lambda}_\Phi x'$ and $\lambda \in L(x)$;
3. $(q, x) \xrightarrow{\varepsilon} (q', x)$ iff $q \xrightarrow{\lambda}_A q'$ and $\lambda \notin L(x)$.

Theorem

F is opaque w.r.t. (A, Φ) iff $F \times X$ is opaque w.r.t. to $(A \otimes \Phi, \Sigma)$.

Consequence: Problem 3 is PSPACE-complete.

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Checking Opacity for Finite State Filters

Opacity for Finite State Filters

Input: A, F, Φ a finite state filter.
Problem: Is F opaque w.r.t. (A, Φ) ?

To check opacity, build a product $A \otimes \Phi$

1. initial state (q_0, x_0)
2. $(q, x) \xrightarrow{\lambda} (q', x')$ iff $q \xrightarrow{\lambda}_A q'$, $x \xrightarrow{\lambda}_\Phi x'$ and $\lambda \in L(x)$;
3. $(q, x) \xrightarrow{\varepsilon} (q', x)$ iff $q \xrightarrow{\lambda}_A q'$ and $\lambda \notin L(x)$.

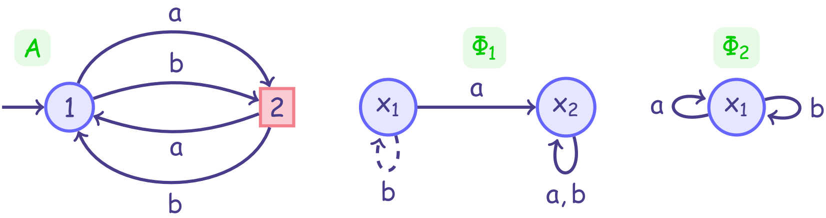
Theorem

F is opaque w.r.t. (A, Φ) iff $F \times X$ is opaque w.r.t. to $(A \otimes \Phi, \Sigma)$.

Consequence: Problem 3 is PSPACE-complete.

How to compare dynamic filters ?

Comparison of Dynamic Filters



- Disabling/Hiding an action costs 1 per time unit (1 t.u. = step of A)
- On input word b^n
  - cost of Φ_1 is n
  - cost of Φ_2 is 0
- Φ_2 is **better** than Φ_1

Need to define a cost measure for dynamic filters

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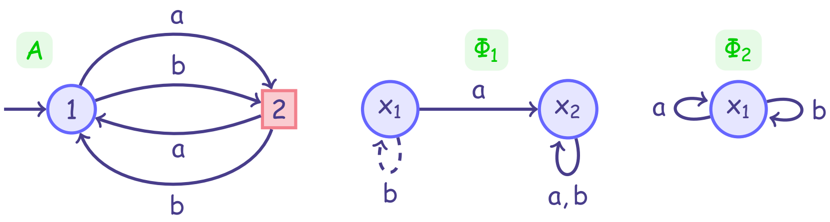

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Computing the Cost of a Given Filter

A run of A: $\rho = q_0 \xrightarrow{\lambda_1} q_1 \quad \cdots \quad q_{n-1} \xrightarrow{\lambda_n} q_n$

$\Phi = (X, x_0, \Sigma, \delta_o, L)$ a filter, and $x_i = \delta_o(x_0, w_i)$ with $w_i = \lambda_1 \cdots \lambda_i$

$C : 2^\Sigma \longrightarrow \mathbb{N}$: Cost of hiding a subset of Σ

Average Cost on a run ρ

$$Cost(\rho, \Phi) = \frac{Cost(\rho)}{|\rho| + 1} = \frac{\sum_{i=0..n} C(\Sigma \setminus L(x_i))}{n+1}.$$

Maximal Cost on runs of A of length n

$$Cost(n, A, \Phi) = \max\{\, Cost(\rho, \Phi) \text{ for } \rho \in Runs^n(A) \,\}.$$

Cost of a pair (A, Φ)

$$Cost(A, \Phi) = \limsup_{n \to \infty} Cost(n, A, \Phi)$$

Theorem

For finite state filters, $Cost(A, \Phi)$ can be computed in PTIME.

Use Karp's Maximum Mean-weight Cycle Algorithm [Karp, 1978]

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Bounded Cost Filter

Problem 4: Bounded Cost Filter

Inputs: a NDA $A = (Q, q_0, \Sigma, \delta, F)$ and an integer $k \in \mathbb{N}$.
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(B) If the answer to (A) is "yes", compute a witness filter.

Steps to solve Problem 4

- Step 1: compute the most permissive filter MPΦ see Problem 5 next
- Step 2: check wether some filter in MPΦ costs less than k

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There is finite state most permissive filter (EXPTIME) for A.

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Problems 4.(A) and 4.(B) can be solved in EXPTIME.

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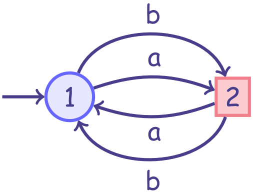
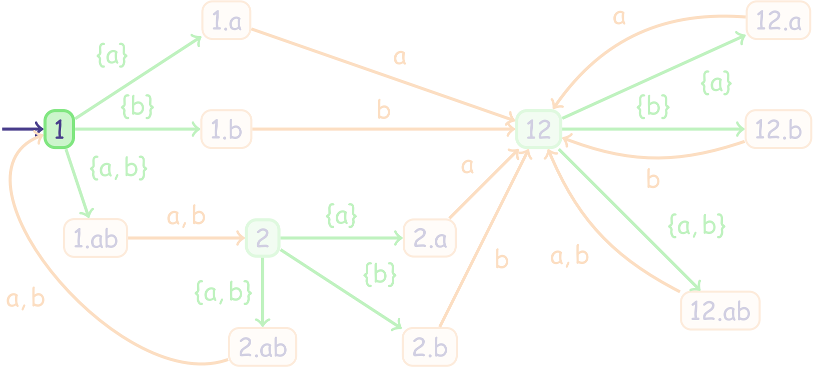
- Player 2 tries to observe F





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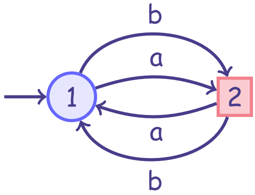

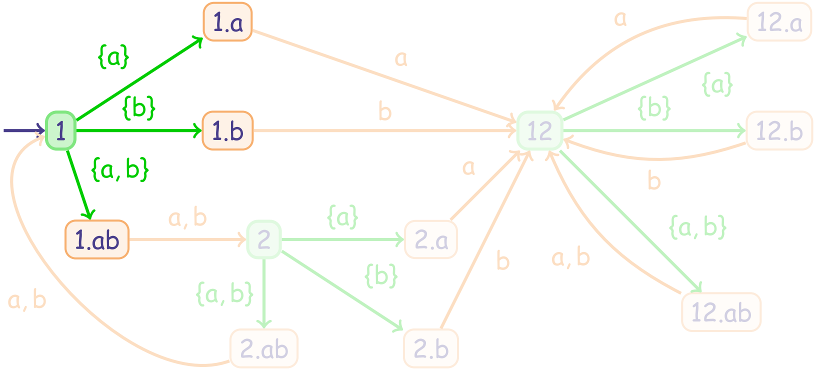

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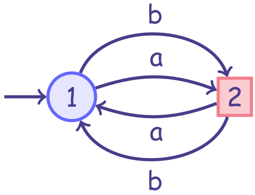

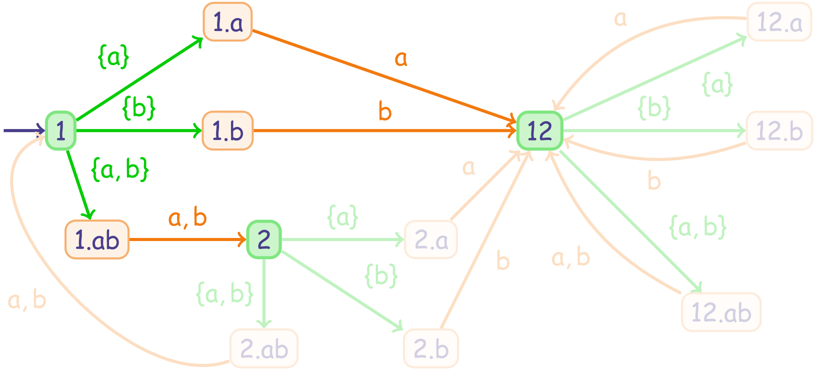

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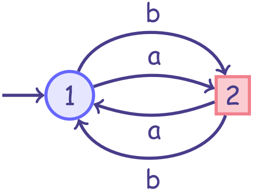

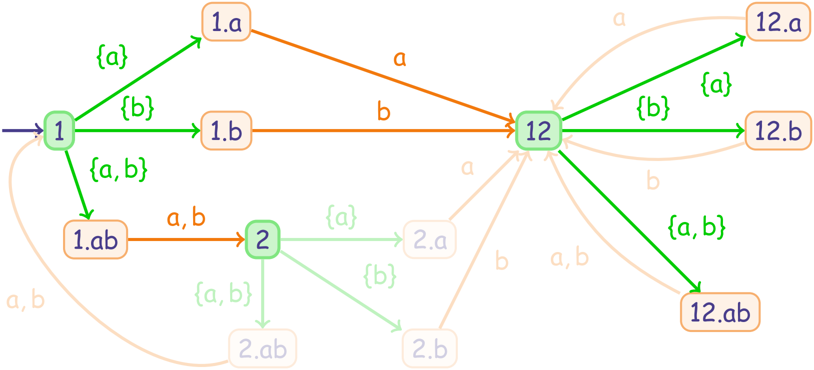

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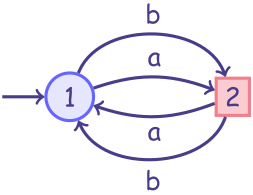

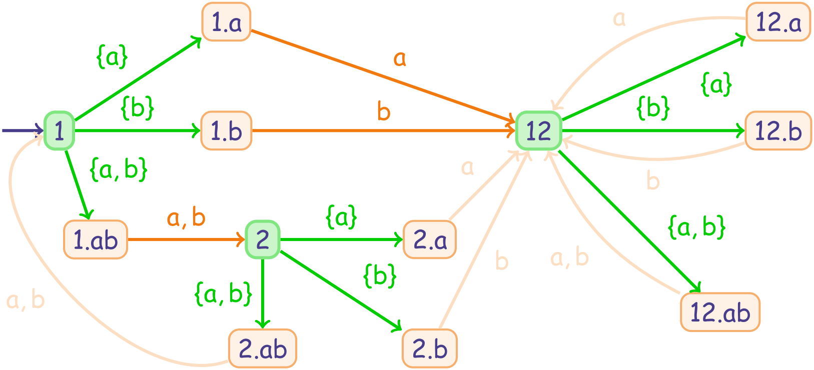

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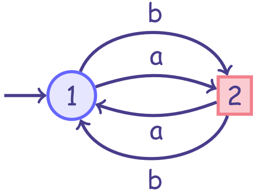

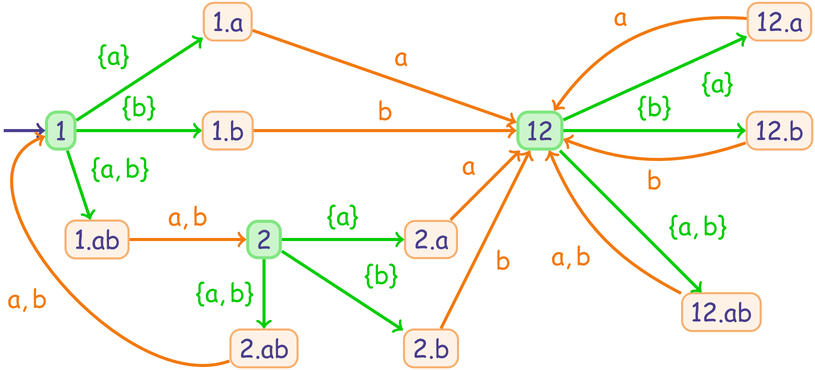

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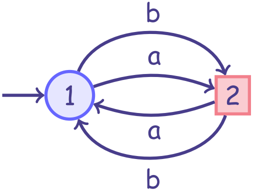

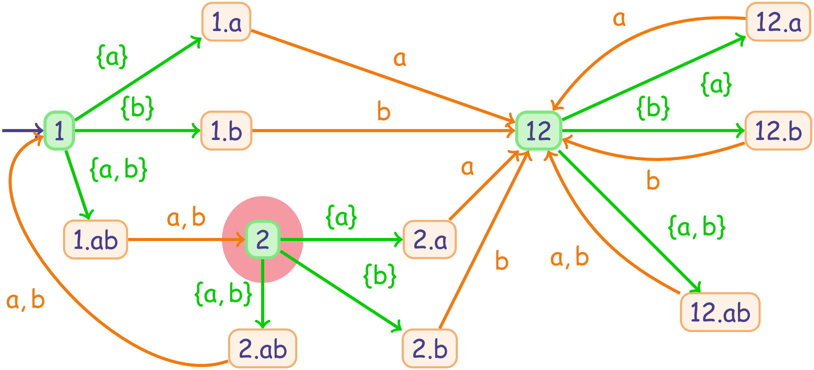

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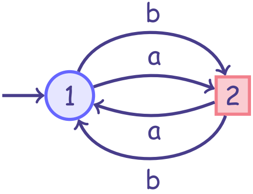

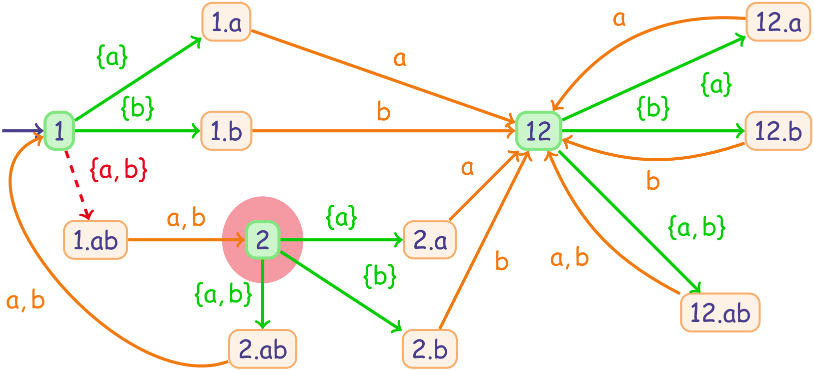

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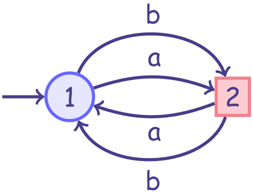
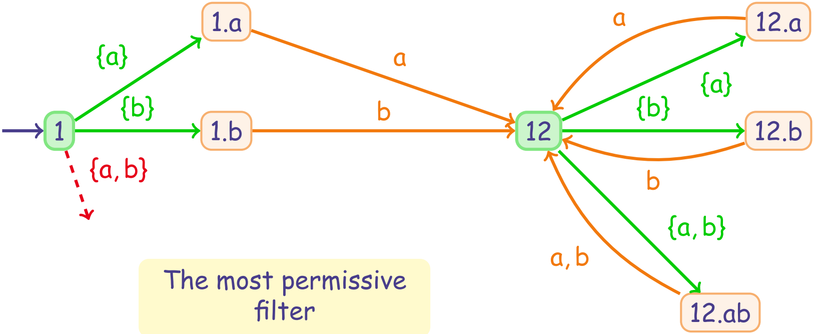




The most permissive filter

Results for Problem 5

Let $G(A, \Sigma)$ be the game defined previously.

Theorem

- if Φ is a filter s.t. F is opaque w.r.t. (A, Φ) then there is a corresponding winning strategy $f(\Phi)$ for Player 1 in $G(A, \Sigma)$
- if f is a winning strategy for Player 1 in $G(A, \Sigma)$, there is a corresponding filter $\Phi(f)$ s.t. F is opaque w.r.t. $(A, \Phi(f))$

Known Result:
There is a memoryless most permissive strategy for any safety game.

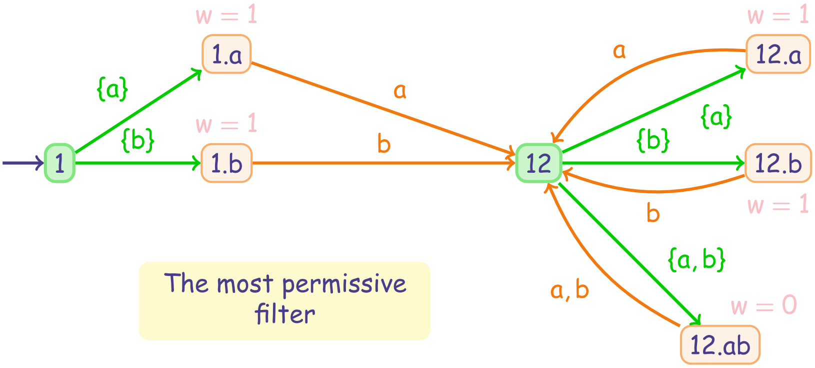
Theorem

There is a finite memory (EXPTIME) most permissive filter $MP\Phi$ for A.

Proof.

$G(A, \Sigma)$ has size exponential in A, Σ.
Solving safety games can be done in linear time. □

Optimal Dynamic Filter



- Player 1 chooses what to hide: strategy f
- Player 2 chooses an action
- Add weight on Player 1's choices
- Player 1 playing f and Player 2 produce weighted runs $w(\rho)$ and

$$\mathbf{Cost}(\rho, f) = w(\rho)/(|\rho| + 1)$$

- Goal for Player 1: minimize $\limsup_{n\to\infty}\{\, w(\rho)/(|\rho| + 1) \mid \rho \in Runs^n(A)\,\}$

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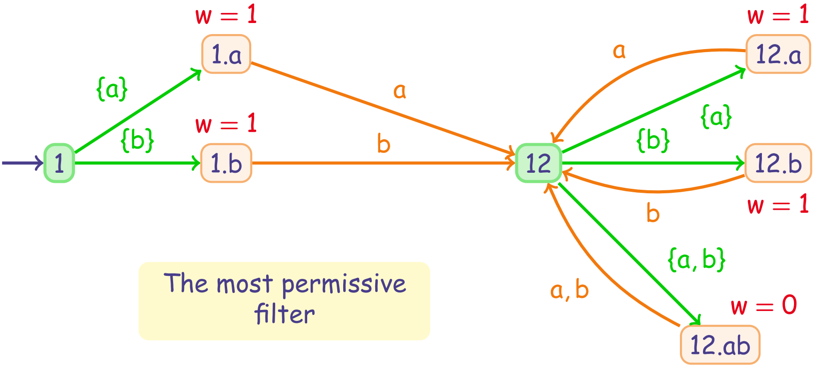

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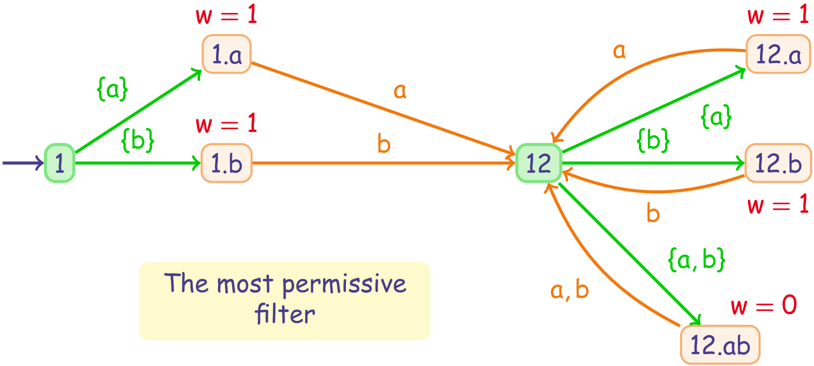

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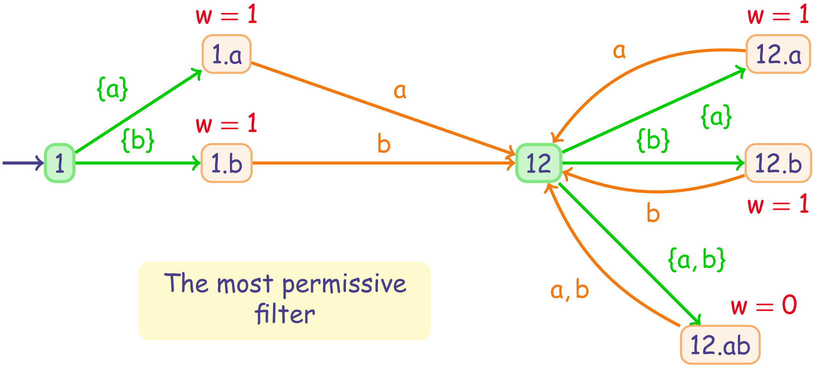

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Mean Payoff Games

[Zwick & Paterson, 1996]

Weighted two-player games

- Each state s has a weight $w(s)$
  alternatively: weight on edges
- Turn-based game
- Goal of the Players:
  - Player 1: minimize $l_0 = \limsup w(\rho)/(|\rho| + 1)$
  - Player 2: maximize $l_1 = \liminf w(\rho)/(|\rho| + 1)$

Results for weighted two-player games

[Zwick & Paterson, 1996]

- There is a value $v \in \mathbb{Q}$ s.t. each player has a memoryless strategy to ensure $l_0 \leq v$ and $l_1 \geq v$
- v can be effectively computed (PTIME)
- Memoryless strategies for both players can be effectively computed

v-Winning Strategy for Player 1 = Optimal filter

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Solution for Problem 4

1. Compute the most permissive filter $MP\Phi$
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3. Use Zwick & Paterson's algorithm to compute the value of the game
4. Compare k to the value of the game

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Problem 4 can be solved in EXPTIME.
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Outline

Results and Future Work

Summary of the Results

- Opacity with dynamic filters
  Secret can also be given by a regular language
- Cost & computation of the cost of a dynamic filter
- Existence & computation of the most permissive filter
- Existence of a finite optimal dynamic observer
- Effective computation of the optimal dynamic observer
- Extended version in [CDM, Tech. Rep., 2009]

Future Work

- Exact complexity of Problem 4 (EXPTIME-hardness)
- Extend to masks (renaming of events)
- Add new constraints to increase the Quality of Services
  e.g. availability properties
- Implement the algorithms

Results and Future Work

Summary of the Results

- Opacity with dynamic filters
  Secret can also be given by a regular language
- Cost & computation of the cost of a dynamic filter
- Existence & computation of the most permissive filter
- Existence of a finite optimal dynamic observer
- Effective computation of the optimal dynamic observer
- Extended version in [CDM, Tech. Rep., 2009]

Future Work

- Exact complexity of Problem 4 (EXPTIME-hardness)
- Extend to masks (renaming of events)
- Add new constraints to increase the Quality of Services
  e.g. availability properties
- Implement the algorithms

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