Predictability of Event Occurrences in Timed Systems

Franck Cassez

NICTA and UNSW, Australia

Alban Grastien

NICTA and ANU, Australia

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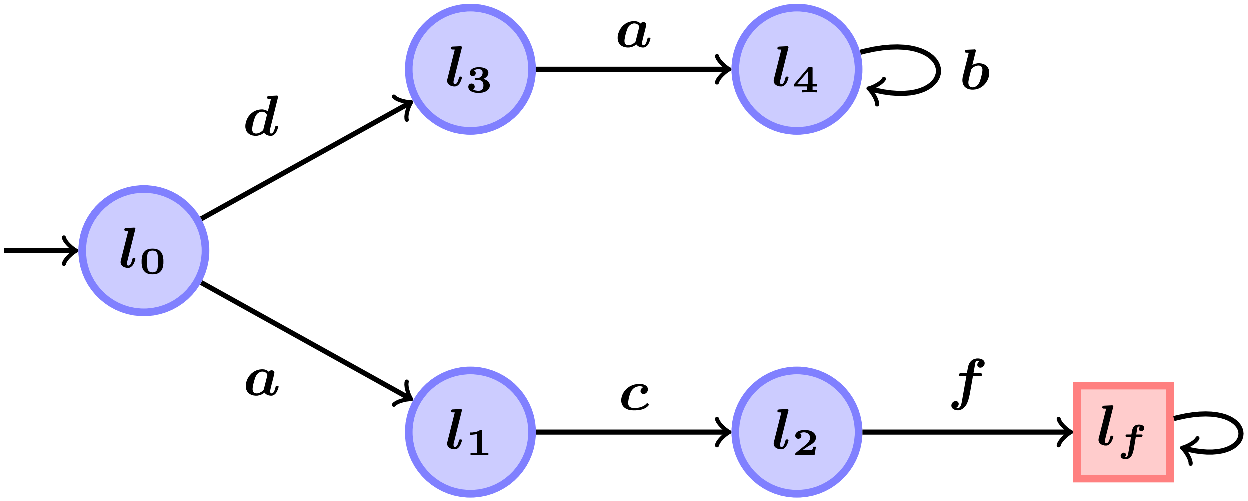
Predictability



$d.a.b^*$

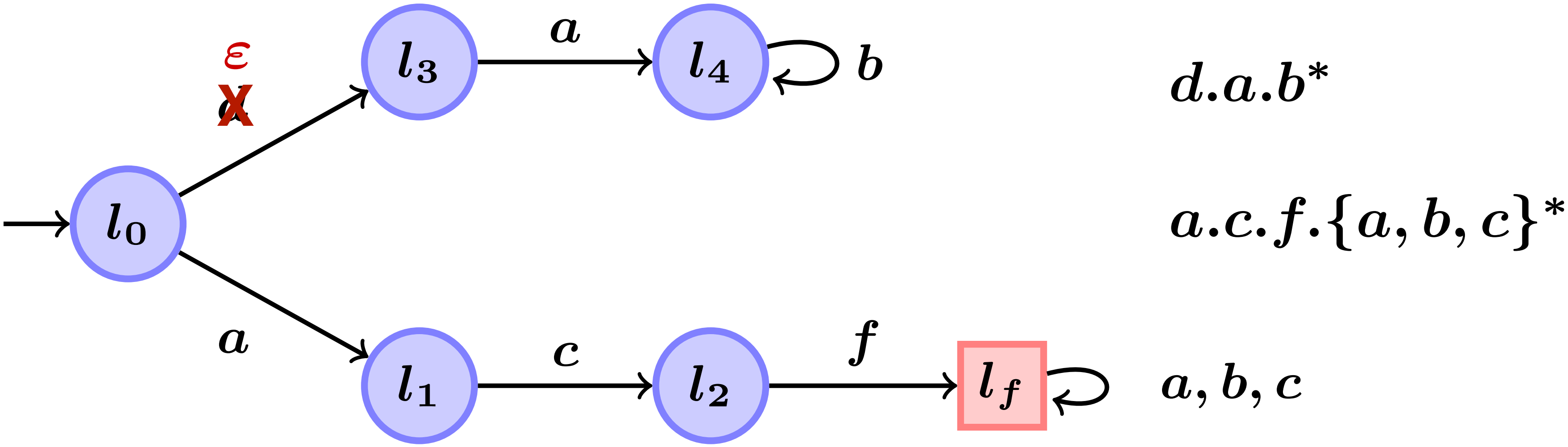
$a.c.f.\{a, b, c\}^*$

a, b, c

System

- generates sequences of events
- model is available

Predictability



System

- generates sequences of events
- model is available
- partially observable

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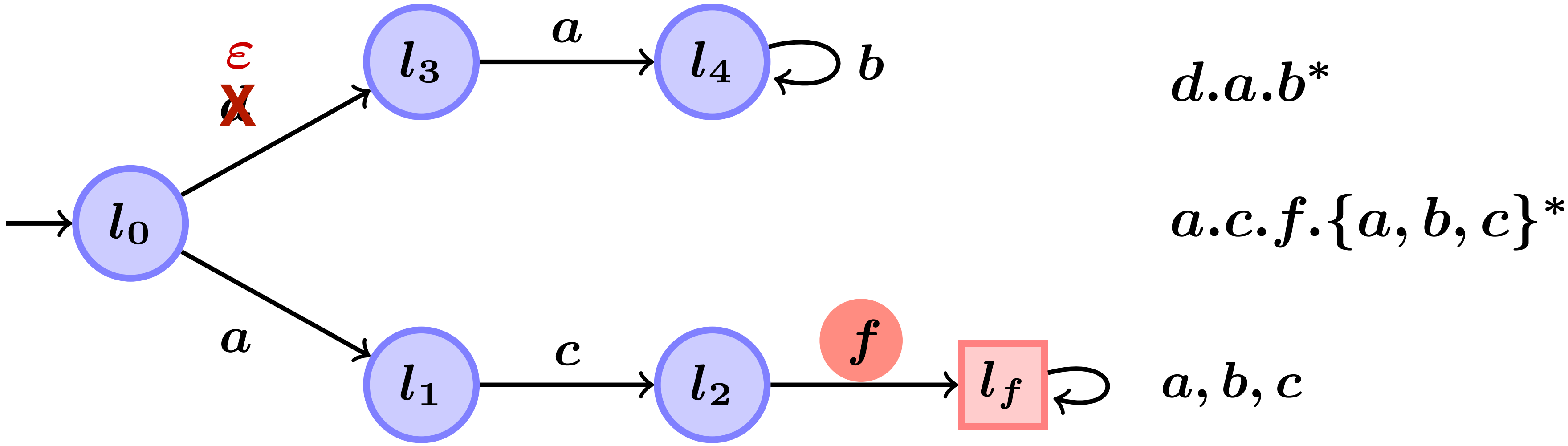

System

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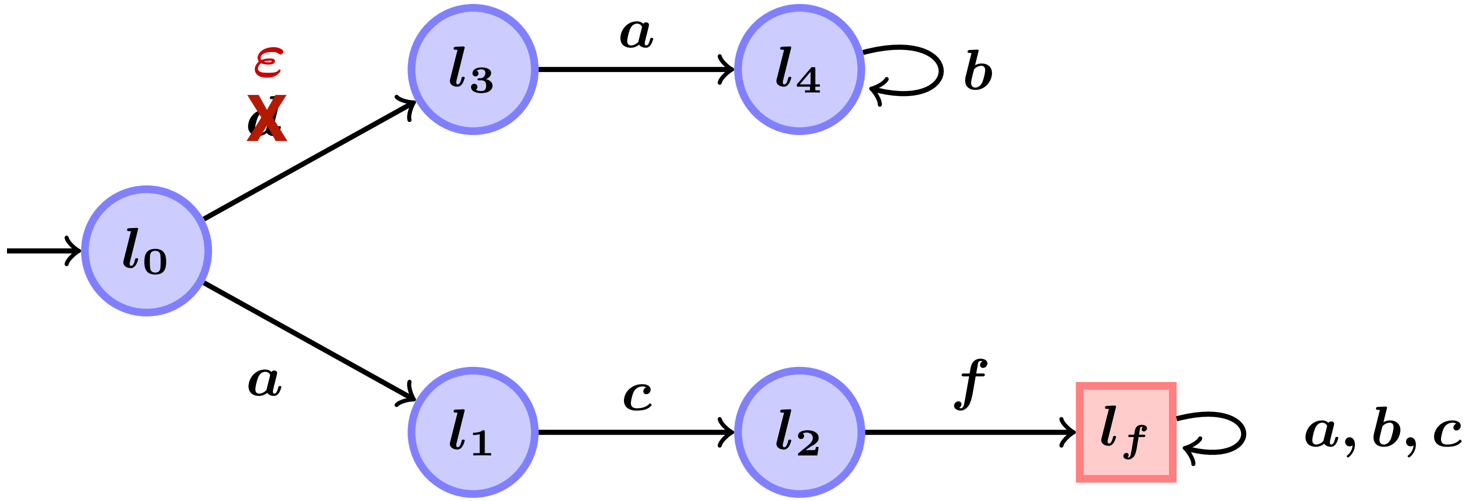
- model is available
- partially observable

Goal: predict event f

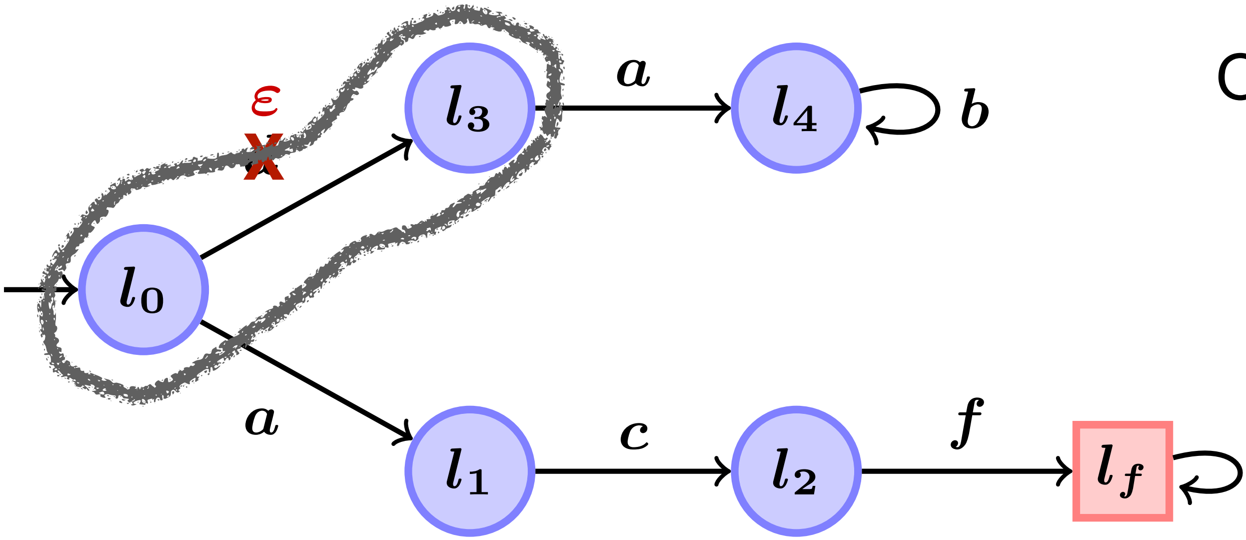
Predictability



Partial observation: projection $\pi(w)$

$$\pi(d.a.b.b) = a.b.b$$

Predictability



Observation

Partial observation: projection $\pi(w)$

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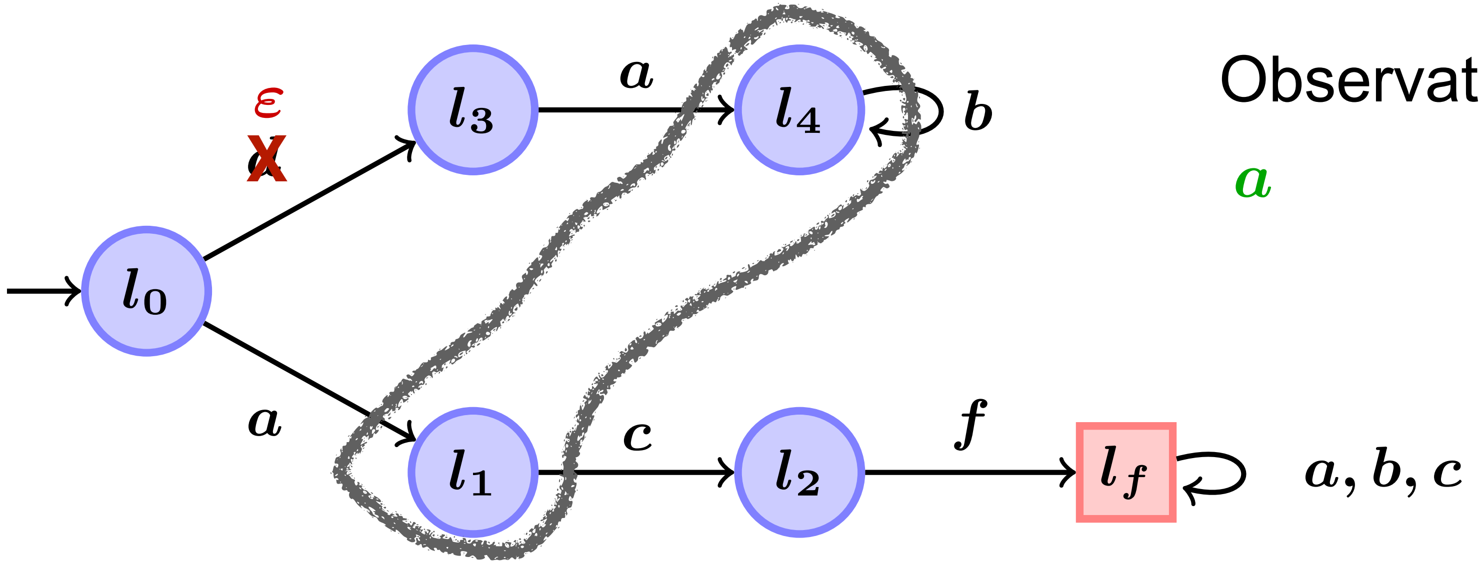

Observation

a

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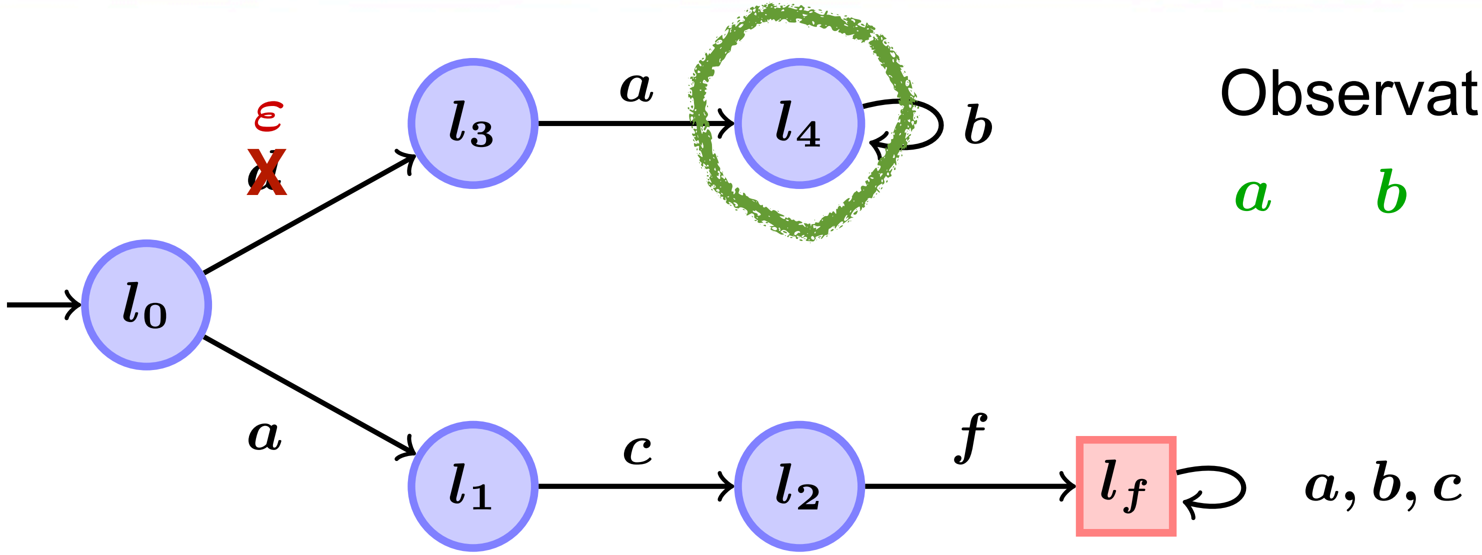
Predictability



Observation

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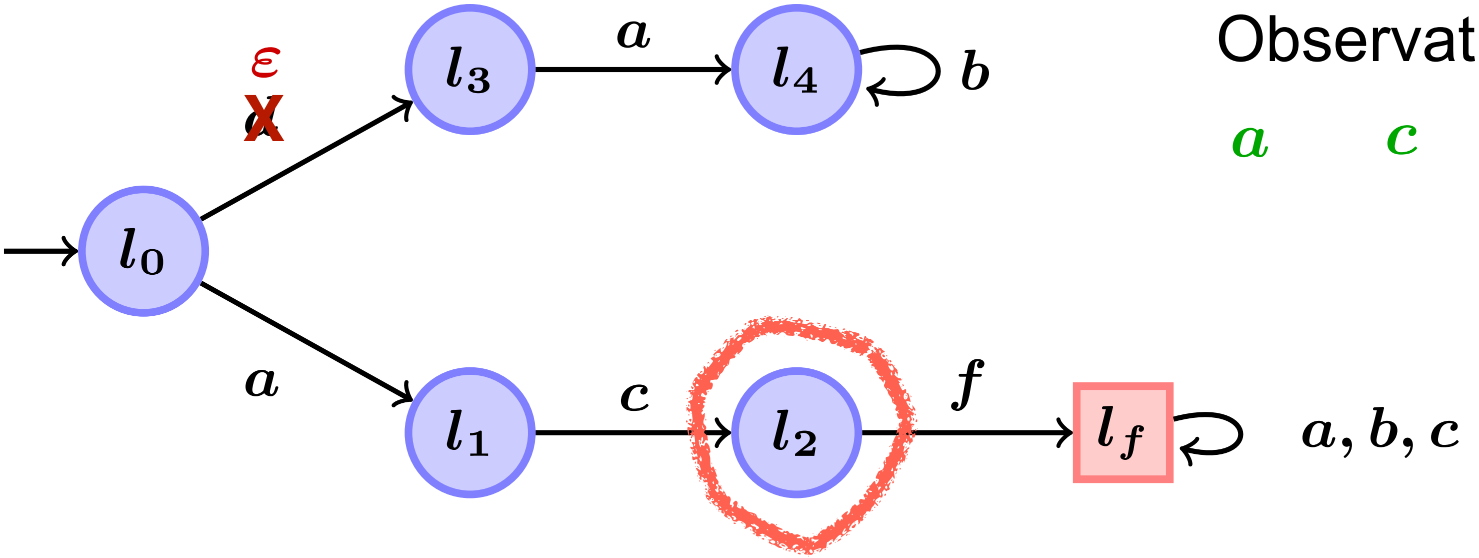
Predictability



Observation

a c

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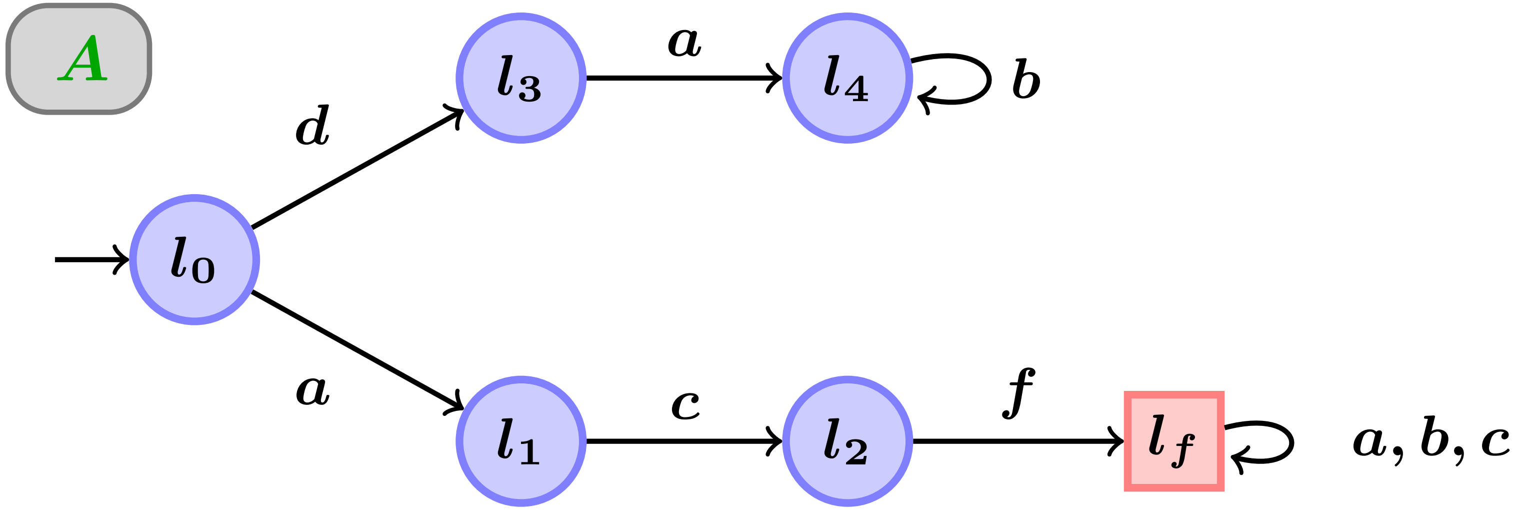
Plan

NICTA

1) Formal simple definition of predictability
   extends previous definition by Genc and Lafortune
   language based and valid for timed systems
2) Predictability for discrete time systems
   polynomial time algorithm
3) Predictability for timed systems
   PSPACE-completeness
4) Sampling predictability
   implementable predictors

Formal Definition of Predictability



$L(A)$ = finite or infinite traces of A $\quad d.a.b^{\omega} \quad a.c.f.b.a$

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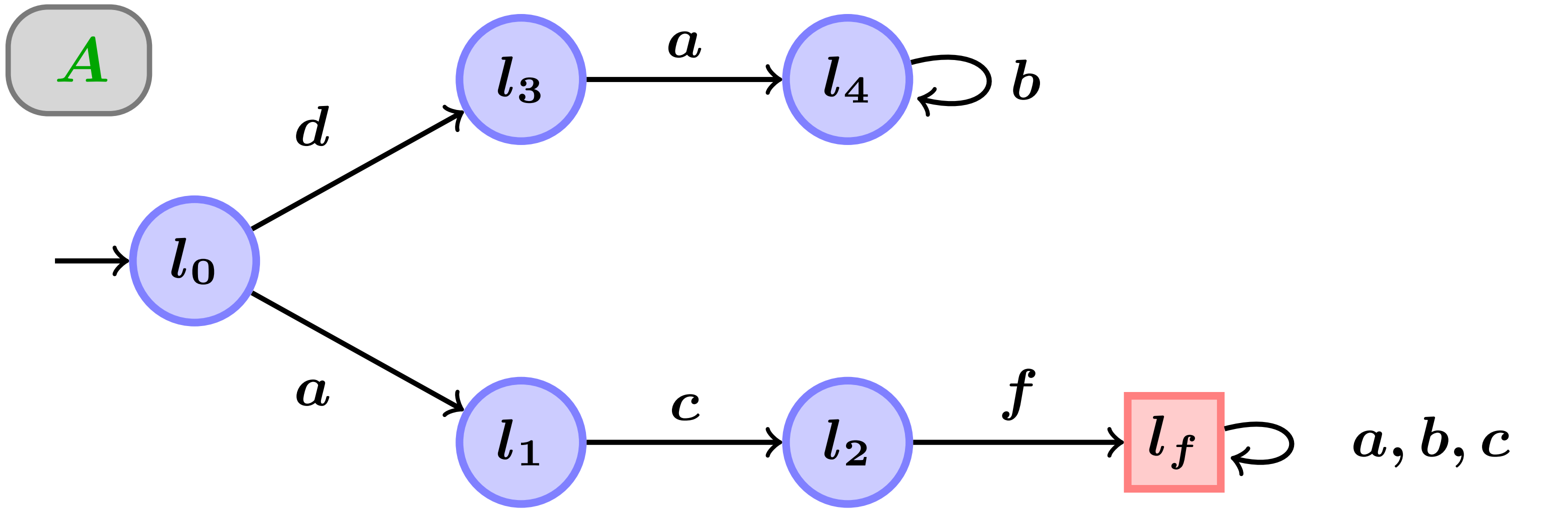

$L(A)$ = finite or infinite traces of A $\quad d.a.b^\omega \quad a.c.f.b.a$

$L^\omega_{\neg f}$ = infinite traces of A with no f $\quad d.a.b^\omega$

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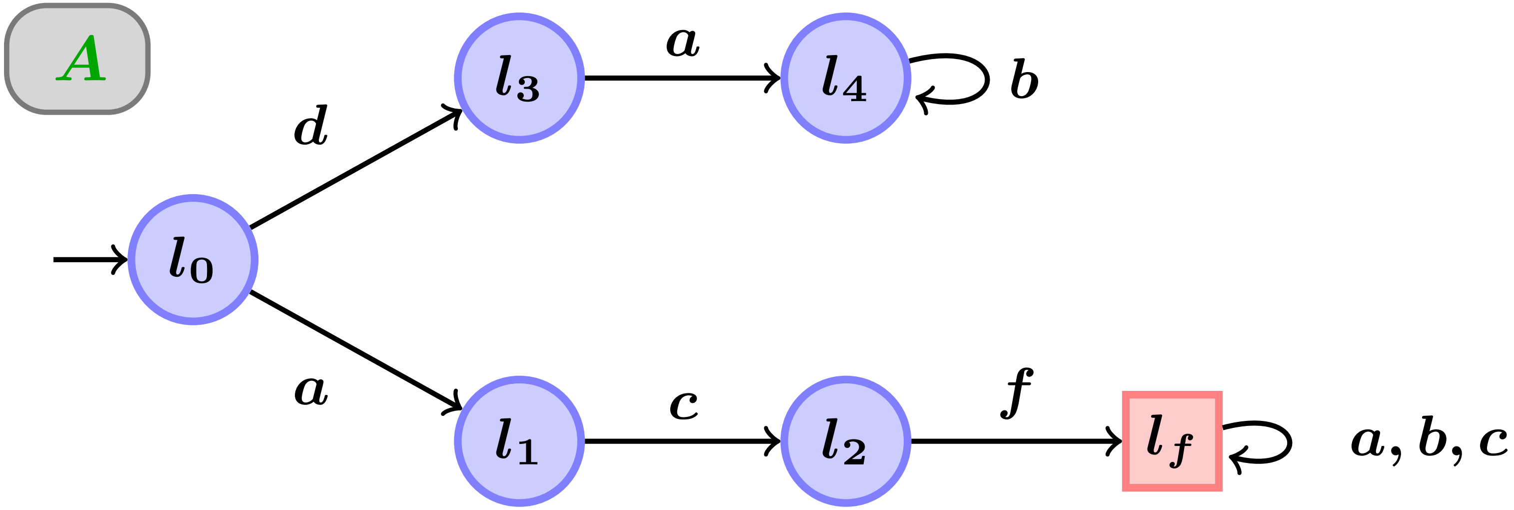

$L(A)$ = finite or infinite traces of A $\quad d.a.b^\omega \quad a.c.f.b.a$

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L_f^{-k} = finite traces w of A with no f

such that $\begin{cases} w.x.f \in L(A) \\ |x| \leq k \end{cases}$

$a.c \in L_f^{-0}$

$a \in L_f^{-1}$

Formal Definition of Predictability

Partial observation: projection $\pi(w)$

(1) k -predictor
(2) CNS for k predictbility and predictability
(3) boxes beige color with equivalence
(4)

k-predictor: mapping P that satisfies:

$$\begin{cases} \forall w \in prefix(L^{\omega}_{\neg f}), P(\pi(w)) = 0 \\ \forall w \in L_f^{-k}, P(\pi(w)) = 1 \end{cases}$$

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k-predictability = existence of a k-predictor

predictability = $\exists k$ such that k-predictable

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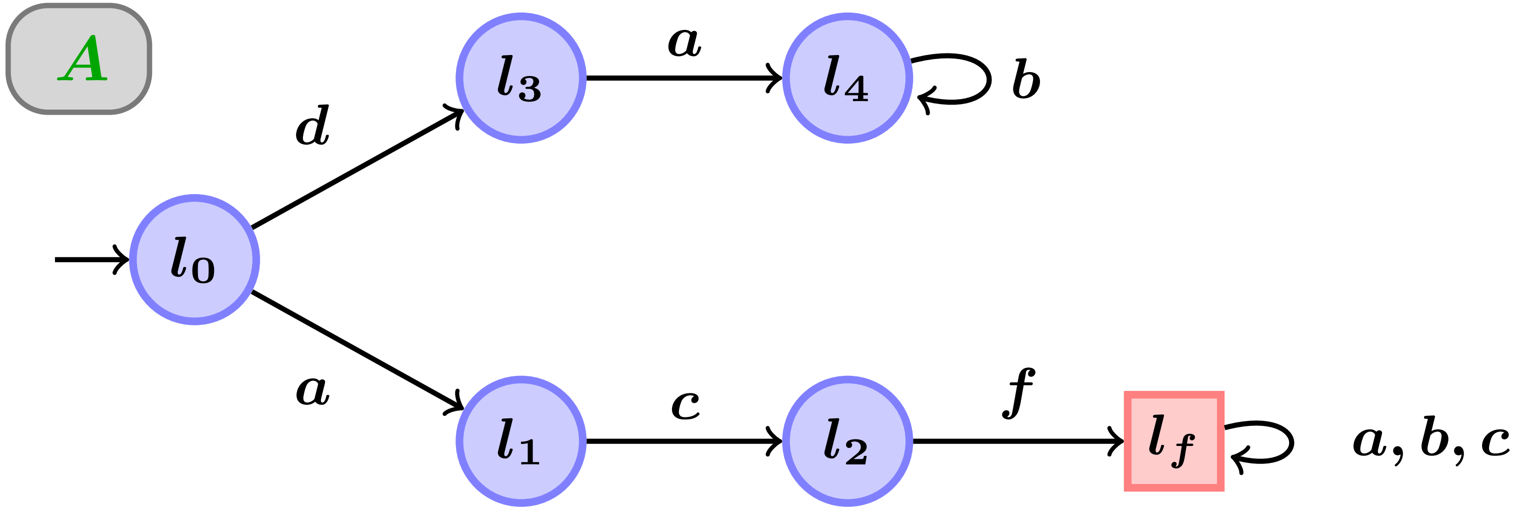
k-predictability $\iff$ $\pi(prefix(L^{\omega}_{\neg f})) \cap \pi(L_f^{-k}) = \emptyset$

predictability $\iff$ 0-predictability

Genc and Lafortune Predictability



$L_{\neg f} = prefix(L^{\omega}_{\neg f})$ $L_f = \overline{L_f}^{\,0}$

GL-predictability:

$\exists n \in \mathbb{N}, \forall w \in L_f, \exists t \in prefix(w)$ such that $\mathbf{F}(t)$

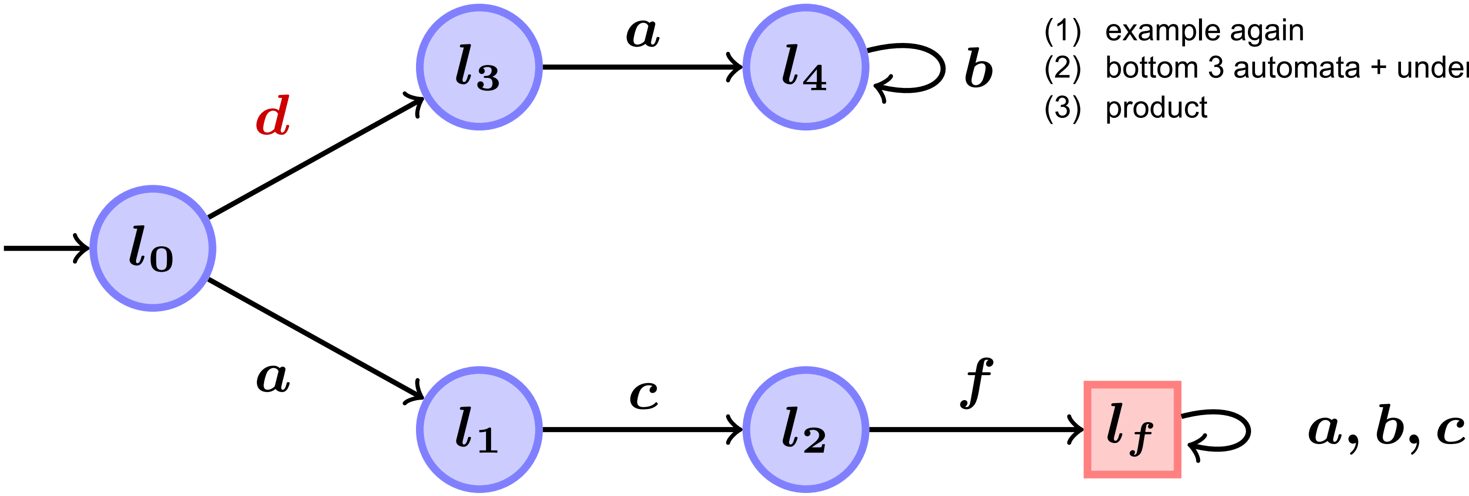
$\mathbf{F}(t) : \forall u \in L_{\neg f}, \forall v = u.x \in \mathcal{L}(A),$

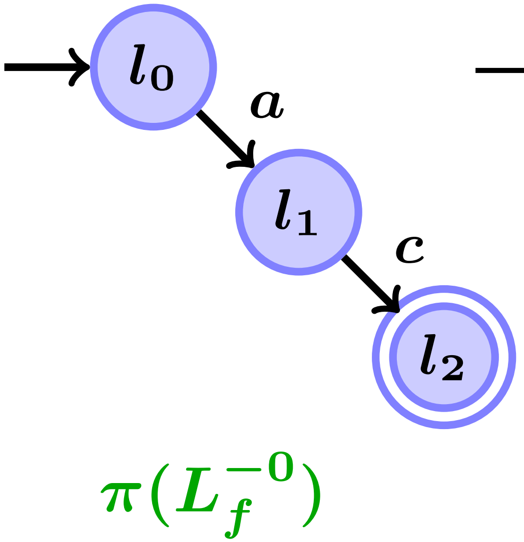
$\pi(u) = \pi(t) \wedge |v| \geq n \implies |v|_f > 0.$

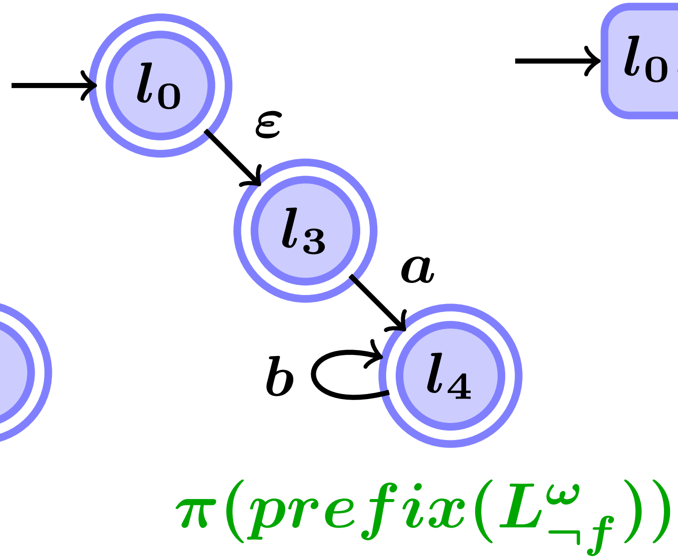
GL-predictability $\iff$ $\mathbf{0}$-predictability

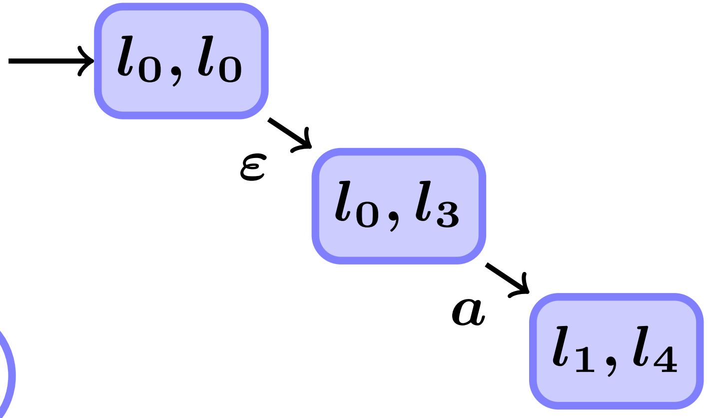
Checking k-Predictability

1. example again
2. bottom 3 automata + under languages
3. product



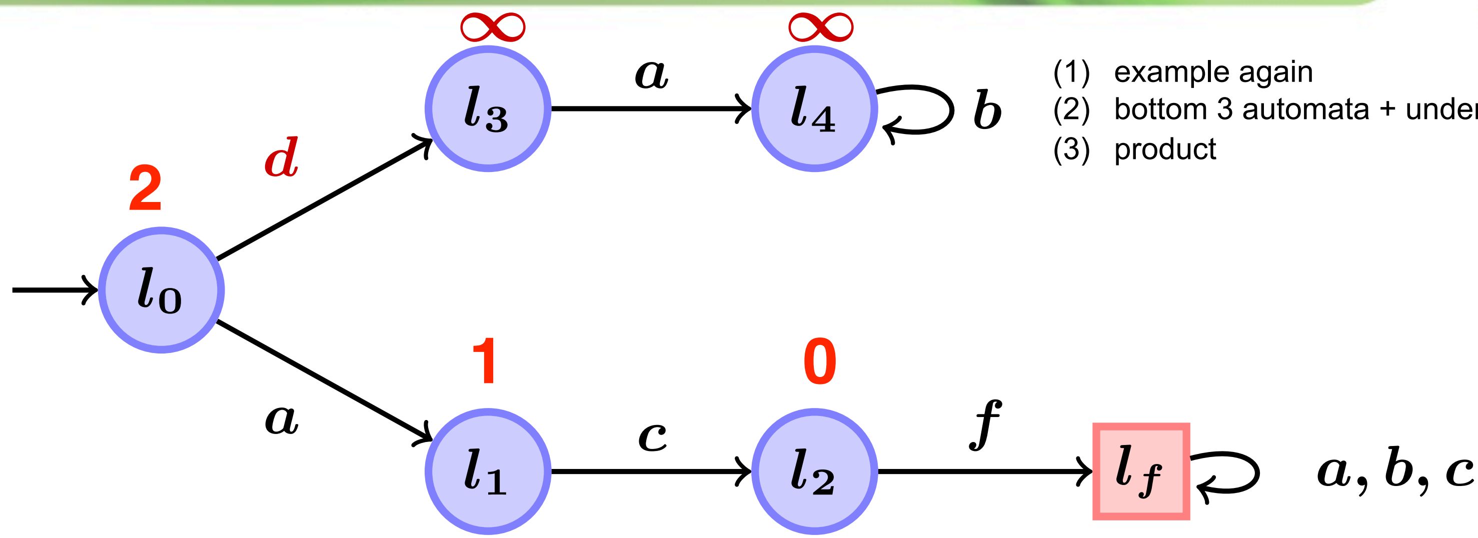


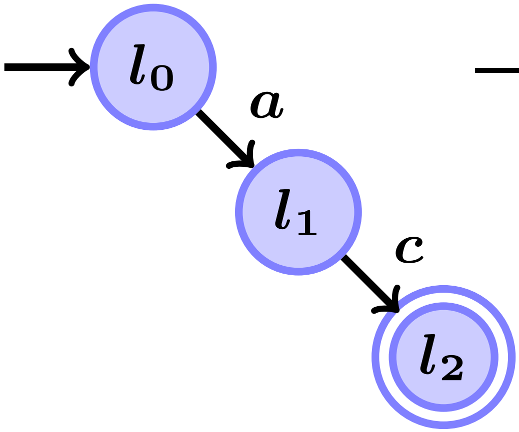




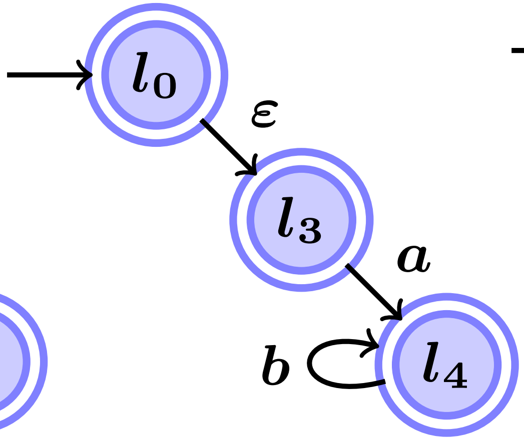
Computing the Maximum k

(1) example again
(2) bottom 3 automata + under languages
(3) product

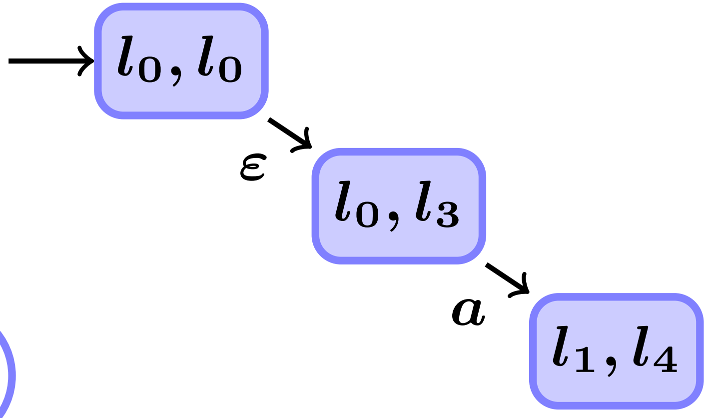




$\pi(L_f^{-0})$



$\pi(prefix(L^{\omega}_{\neg f}))$



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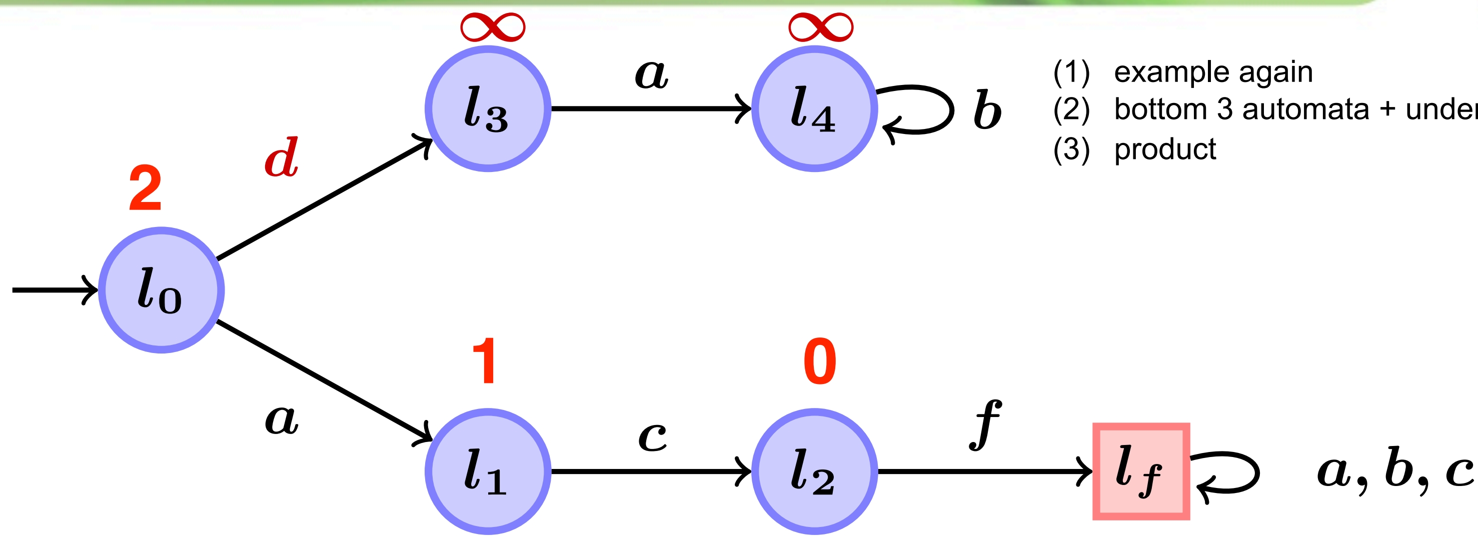
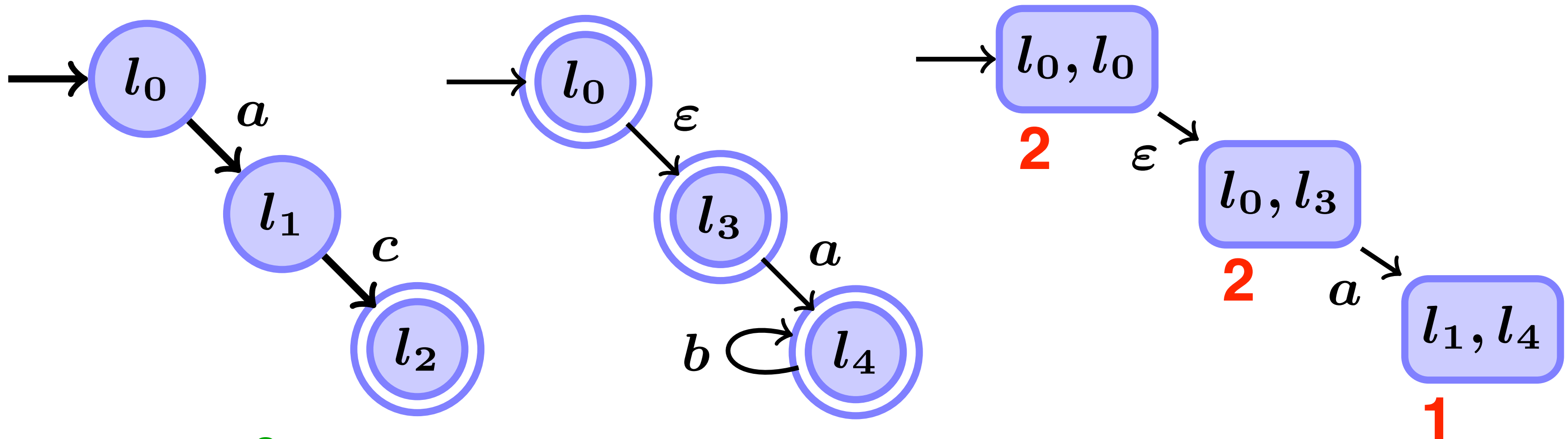




$\pi(L_f^{-0})$ $\pi(prefix(L^{\omega}_{\neg f}))$

Computing the k-Predictor

If k-predictor exists:

$$\begin{cases} \forall w \in prefix(L^{\omega}_{\neg f}), P(\pi(w)) = 0 \\ \forall w \in L_f^{-k}, P(\pi(w)) = 1 \end{cases}$$

and $\pi(prefix(L^{\omega}_{\neg f})) \cap \pi(L_f^{-k}) = \varnothing$

Build a deterministic automaton that accepts $\pi(L_f^{-k})$

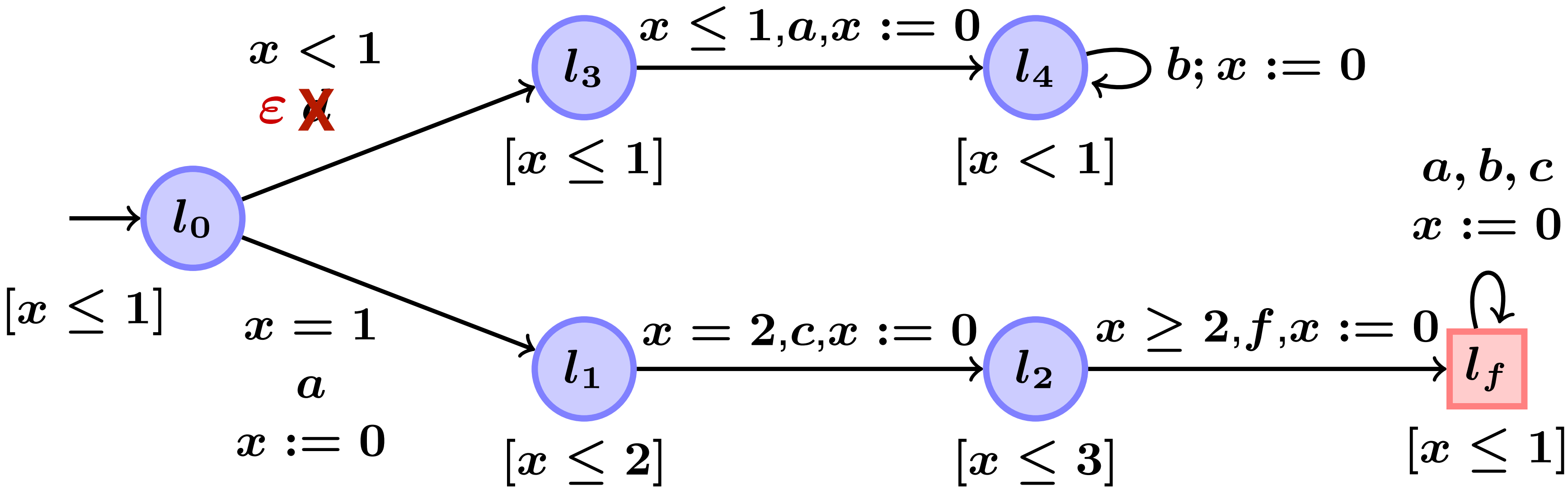
or

on-the-fly determinisation

If A is predictable, there is a finite state predictor

Worst-case: exponential size in A

Predictability for Timed Automata



$L(A)$ = timed traces of A $\quad$ $1\ a\ 2\ c\ 2.5\ f$

projection: $\pi(0.6\ d\ 0.2\ a\ 0.1\ b) = (0.6 + 0.2)\ a\ 0.1\ b$

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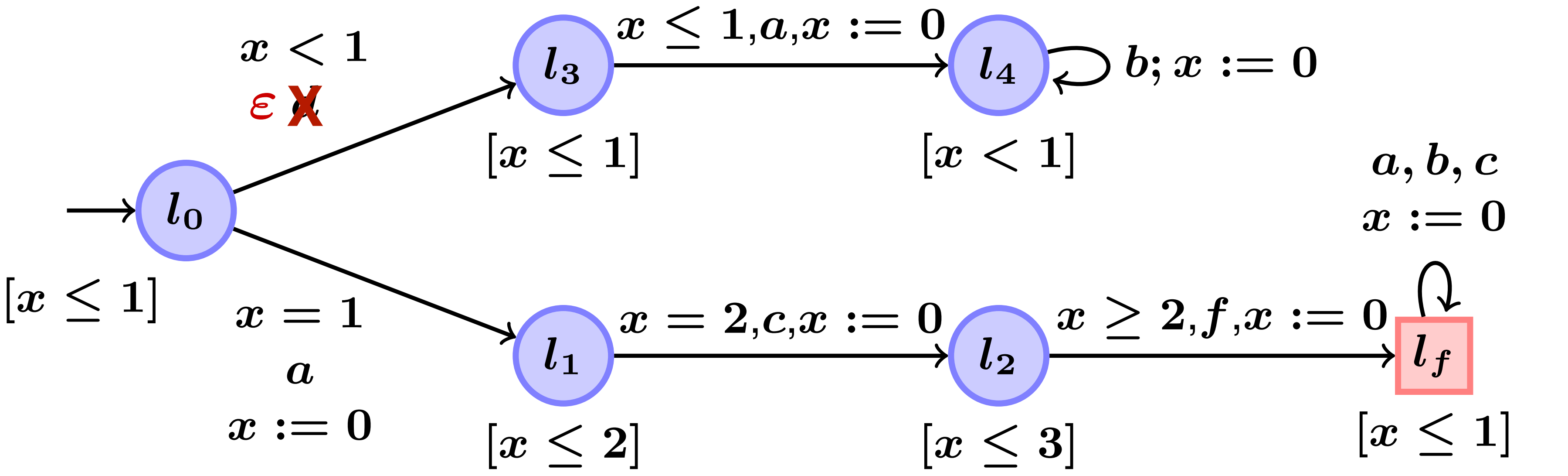

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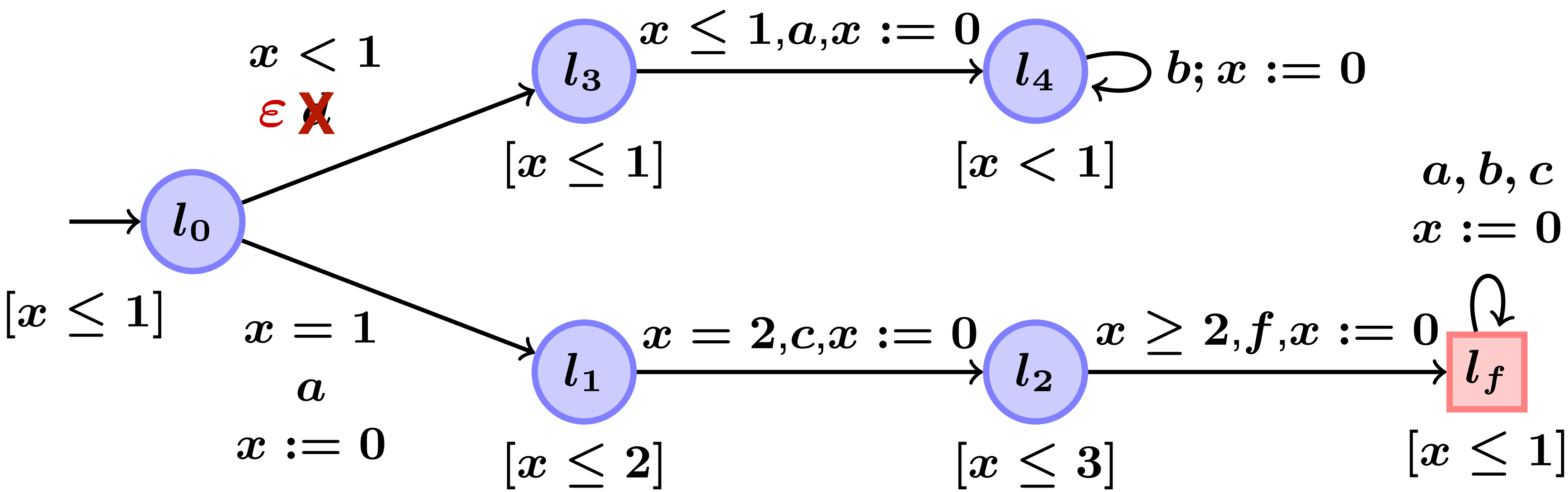

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L_f^{-k} = finite timed traces w of A with no f such that

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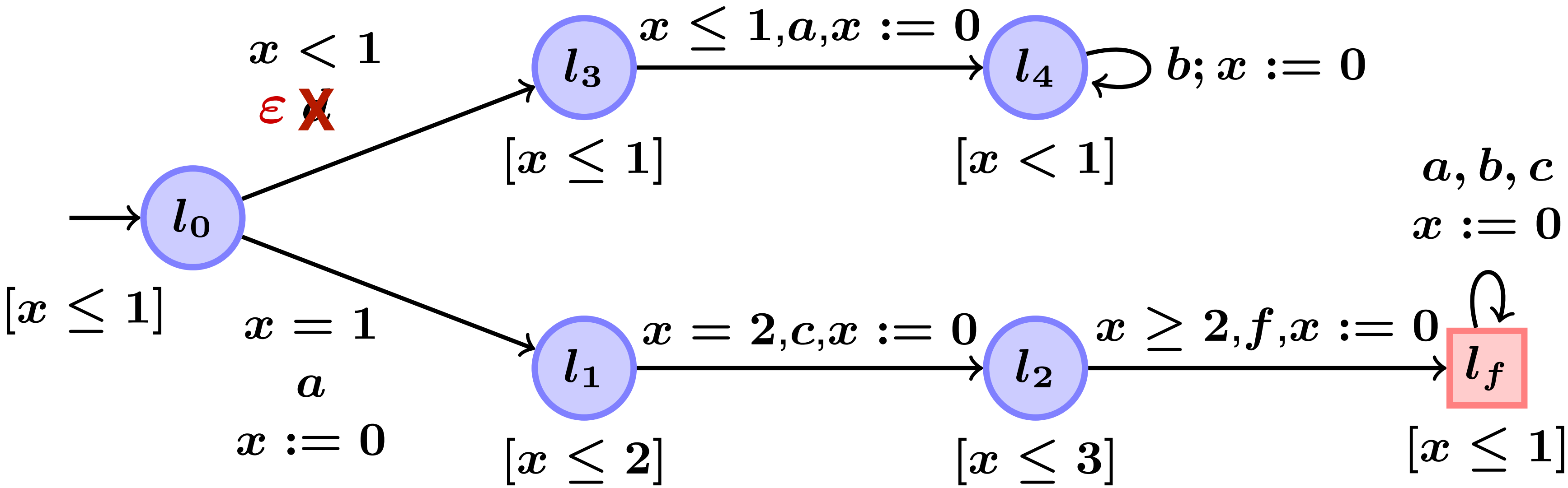
$1\ a\ 2\ c \in L_f^{-3}$

$1\ a\ 2\ c \in L_f^{-2}$

$1\ a\ 2\ c\ 2 \in L_f^{-0}$

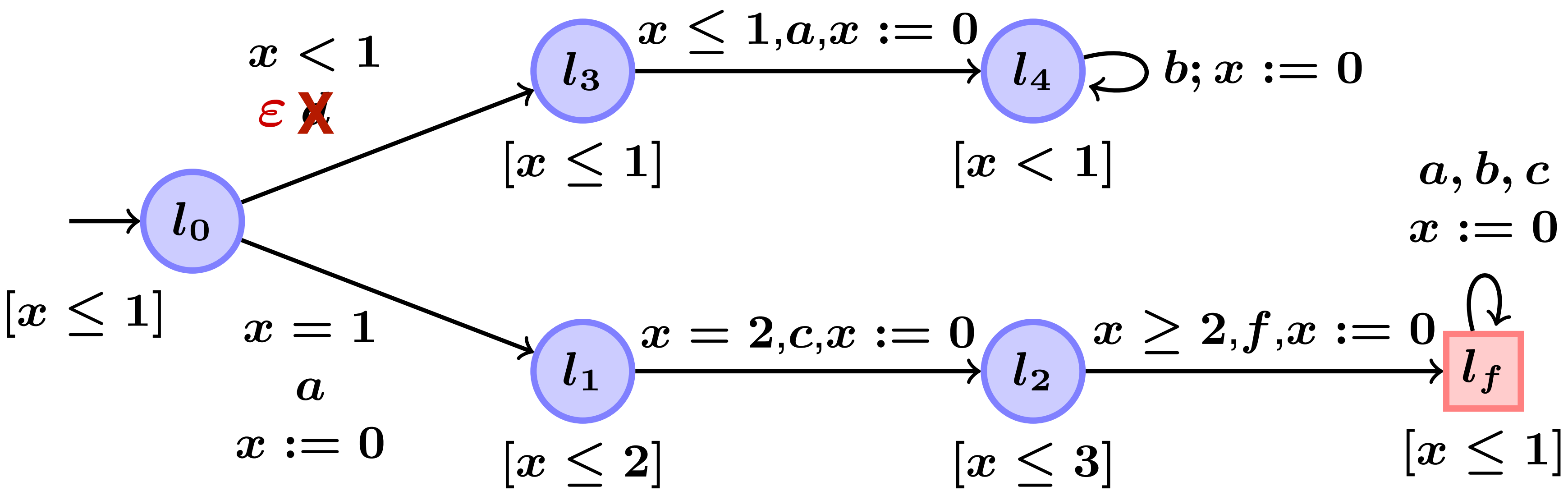
Predictability for Timed Automata



sequences without f

$\delta_d \; d \; \delta_a \; a \; \delta_b \; b \; \cdots$ and $\delta_d + \delta_a + \delta_b < 2$

Predictability for Timed Automata



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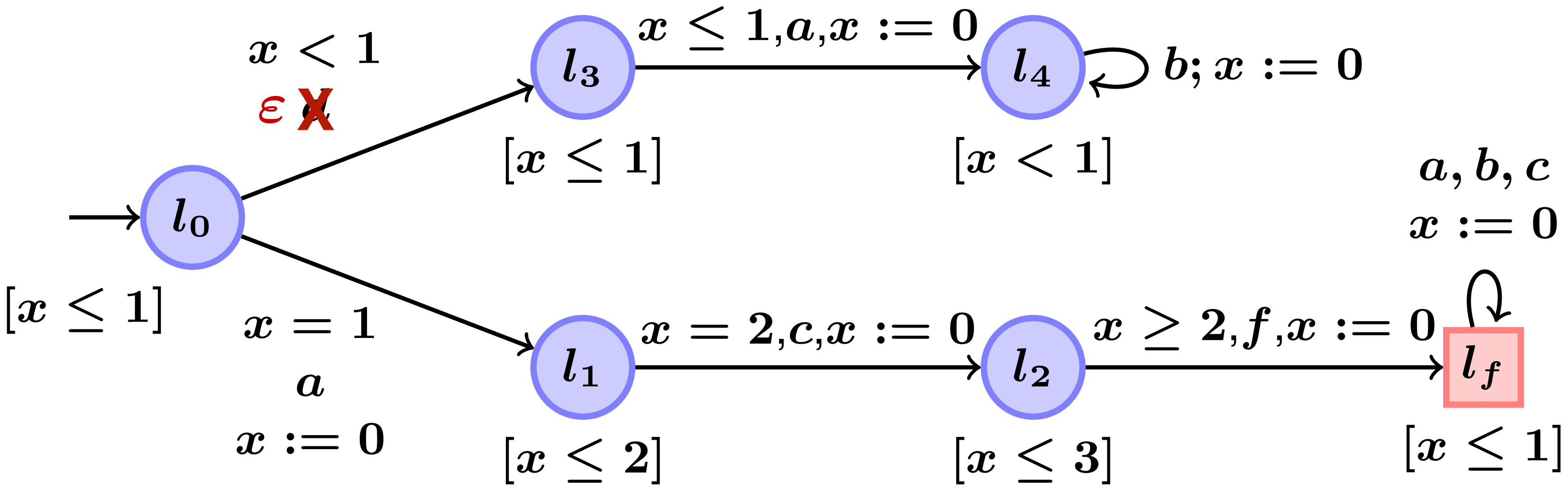

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3-predictable

Checking Predictability for TA

Partial observation: projection $\pi(w)$

k-predictor: mapping P that satisfies:

$$\begin{cases} \forall w \in prefix(L^{\omega}_{\neg f}), P(\pi(w)) = 0 \\ \forall w \in L_f^{-k}, P(\pi(w)) = 1 \end{cases}$$

k-predictability $=$ existence of a k-predictor

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NICTA

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Timed languages

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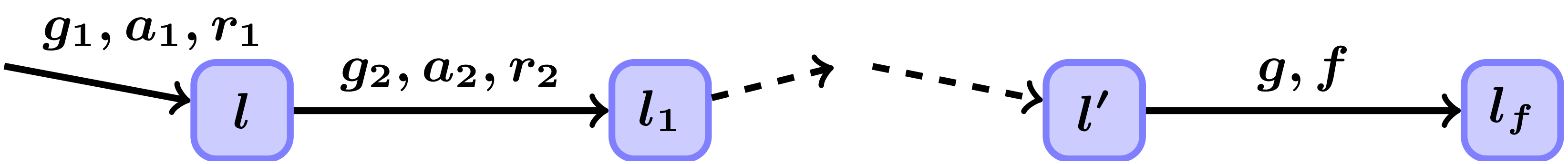

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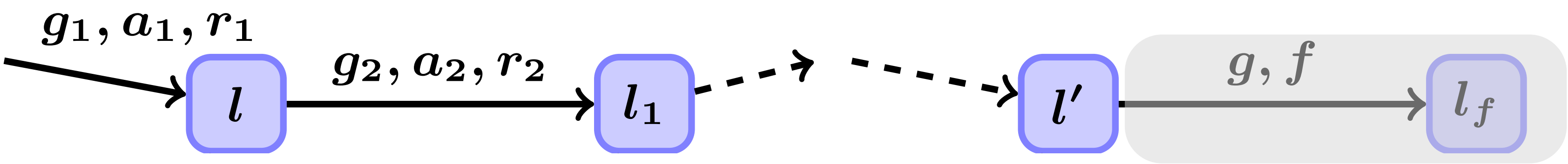

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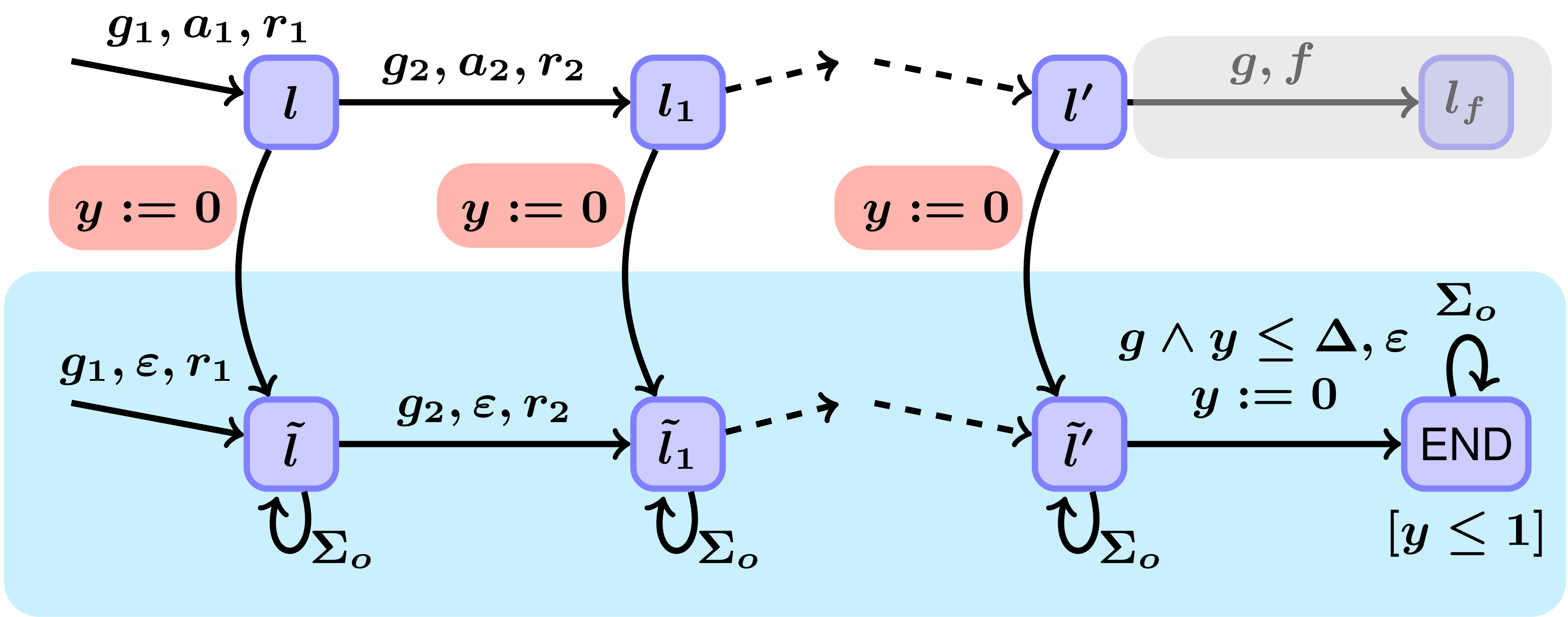

Σ_o = set of observable events

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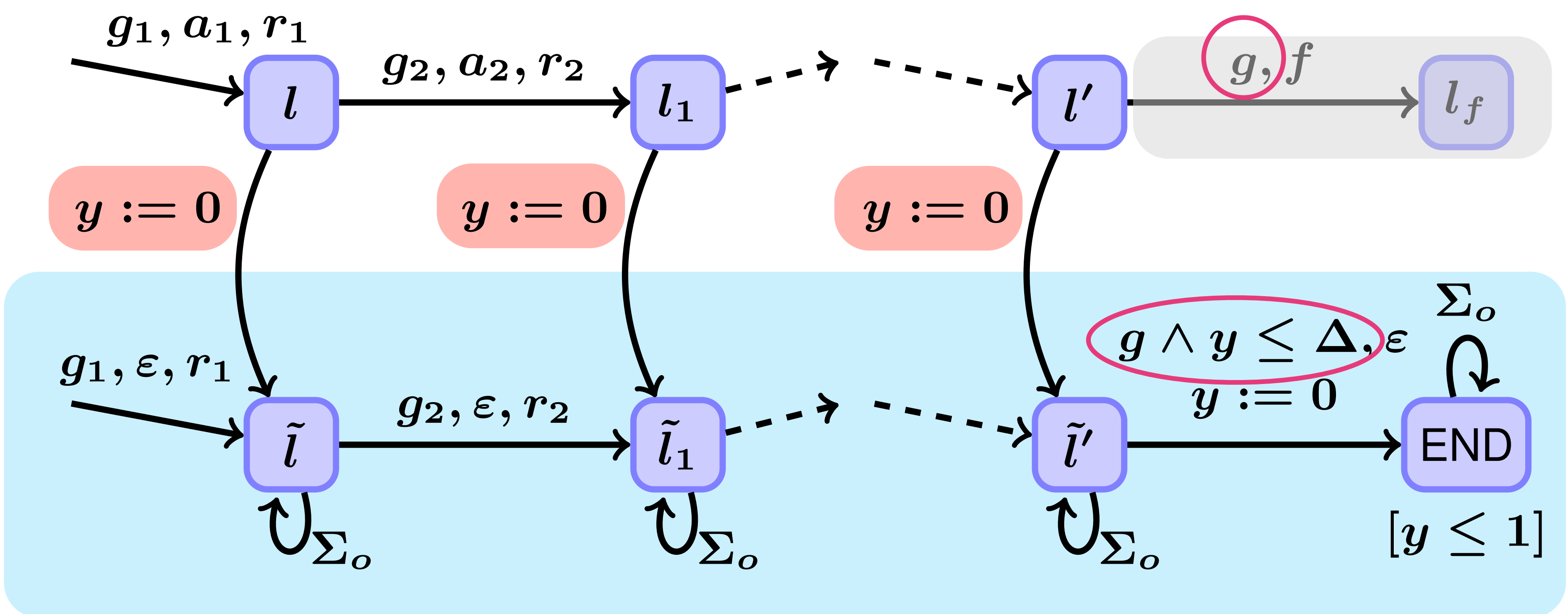

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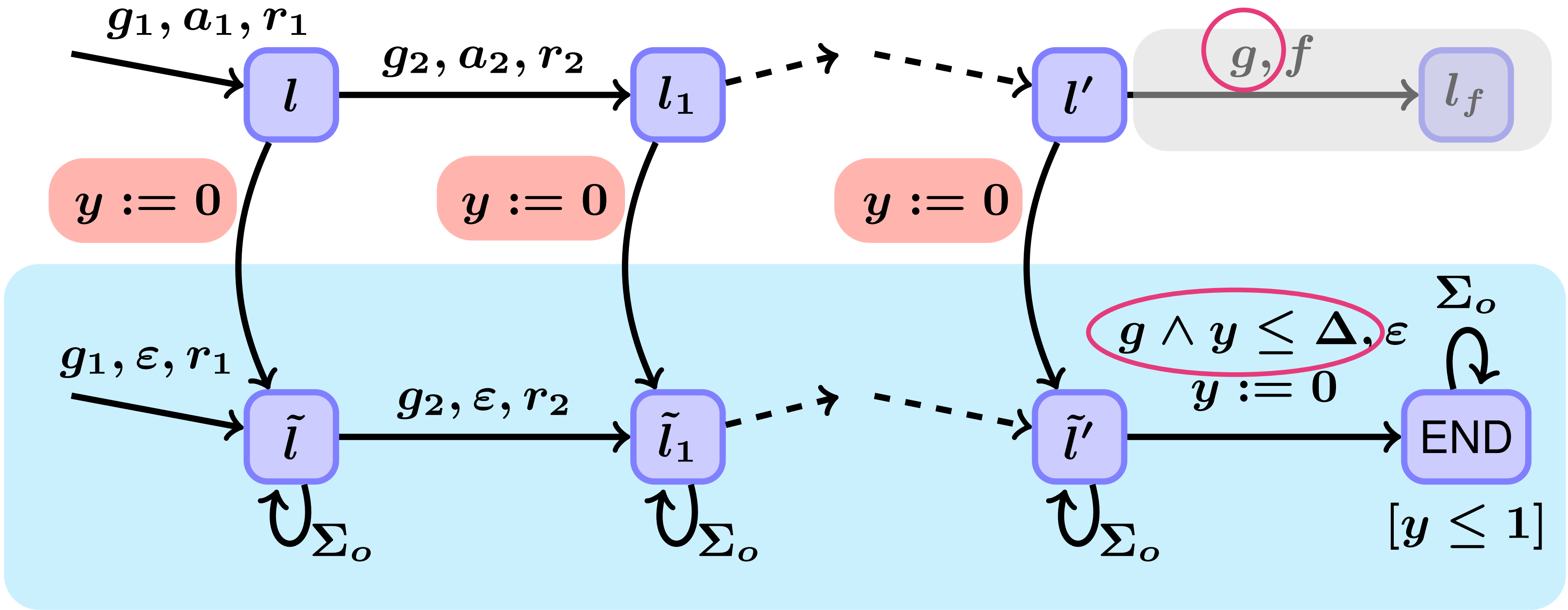


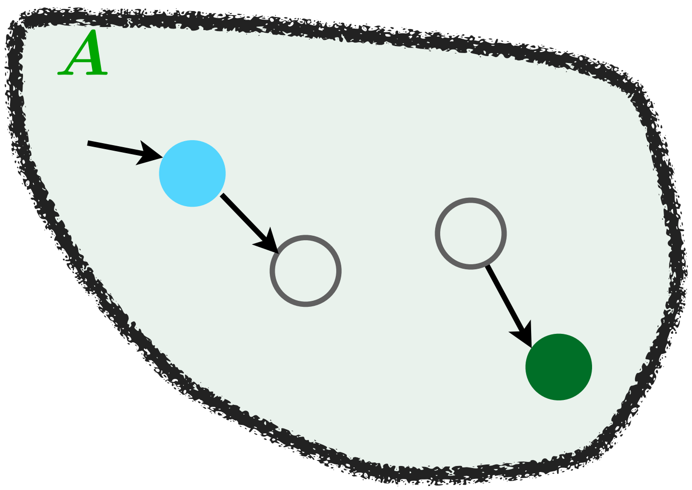
Σ_o = set of observable events

k-predictability $\iff$ $\pi(prefix(L^{\omega}_{\neg f})) \cap \pi(L_f^{-k}) = \varnothing$

$\iff$ $\pi(L^{\omega}_{\neg f}) \cap \pi(L_f^{-k}.(All)^{\omega}) = \varnothing$

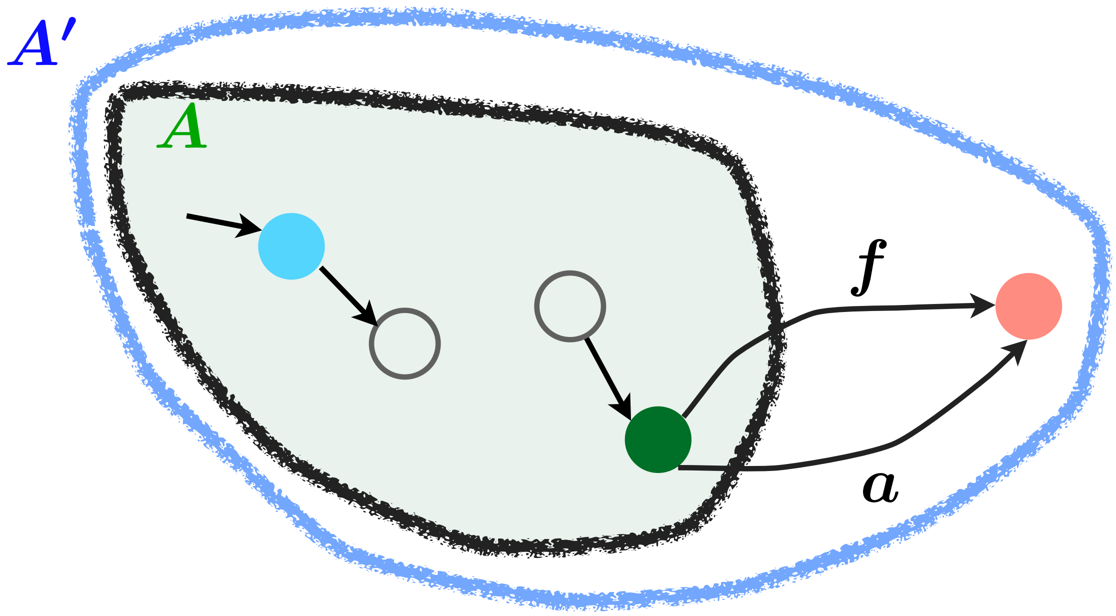
Büchi emptiness for TA: PSPACE-easy

PSPACE-Hardness



Reachability is PSPACE-hard for TA

PSPACE-Hardness



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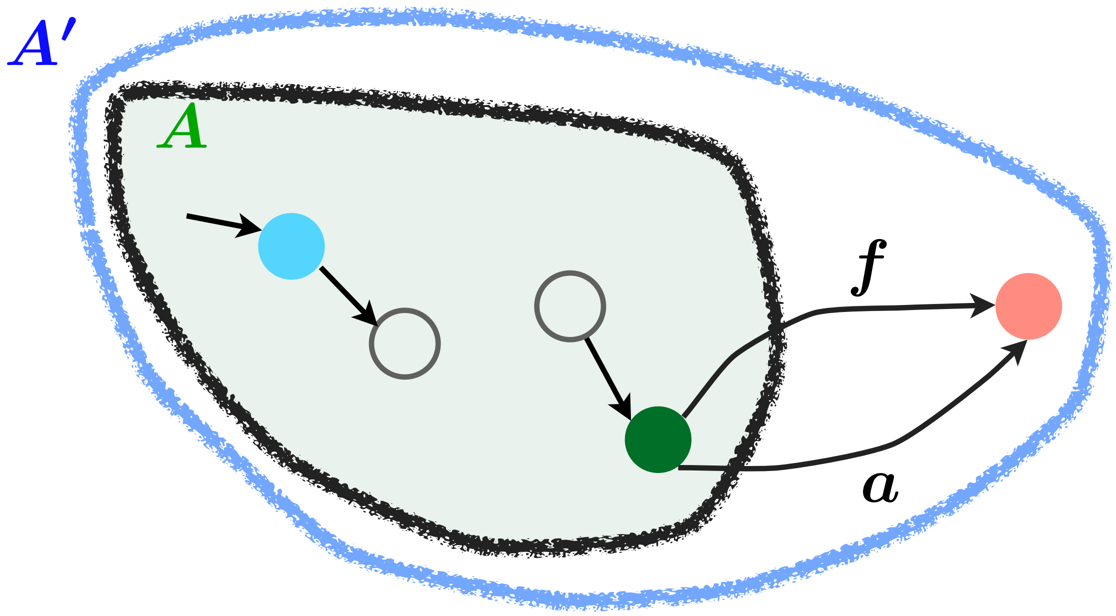

Reachability is PSPACE-hard for TA

● is reachable in A iff A' is predictable.

Predictability is PSPACE-hard

Implementability Issues

Predictor should:

- accept the timed language $\pi(L_f^{-k})$
- be deterministic

Timed Automata are not (always) determinisable

Predictor: online computation of state estimates

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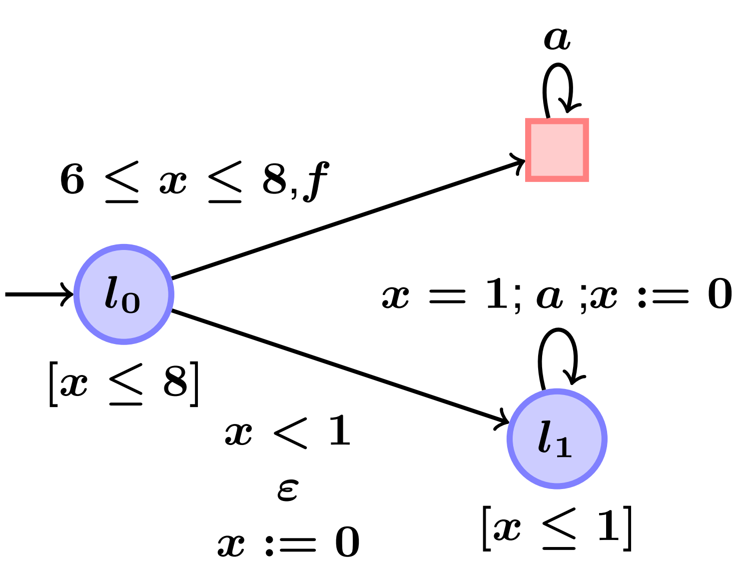

Predictor updates its estimate

- on each observable event
- after a time out

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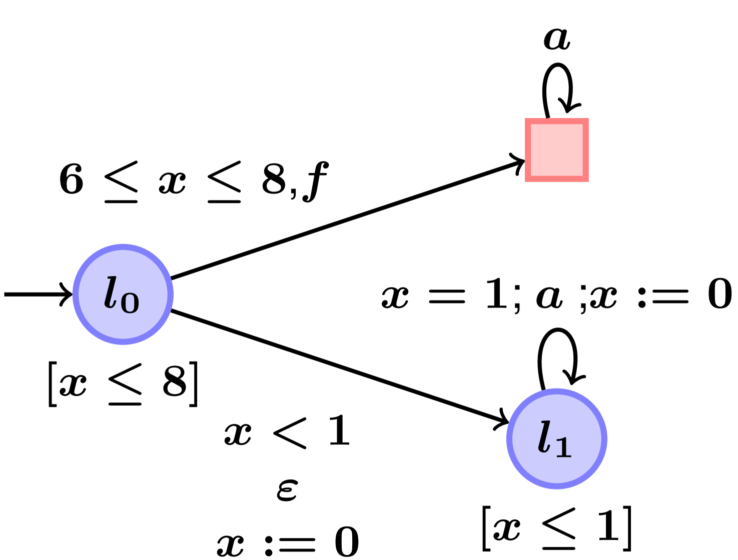

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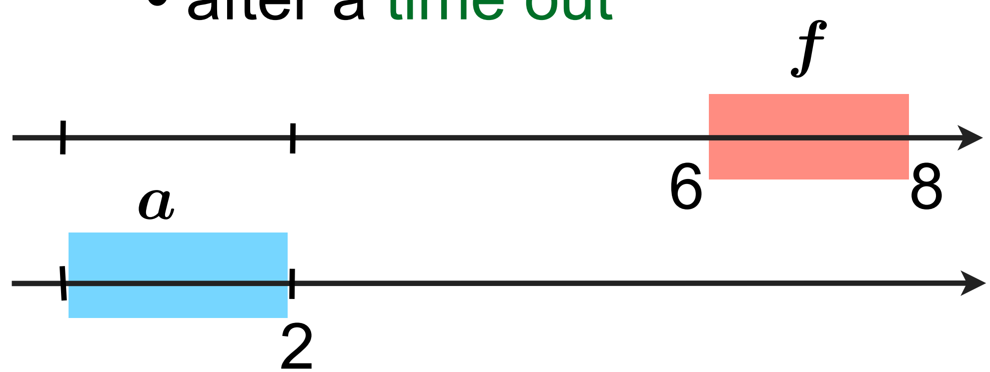

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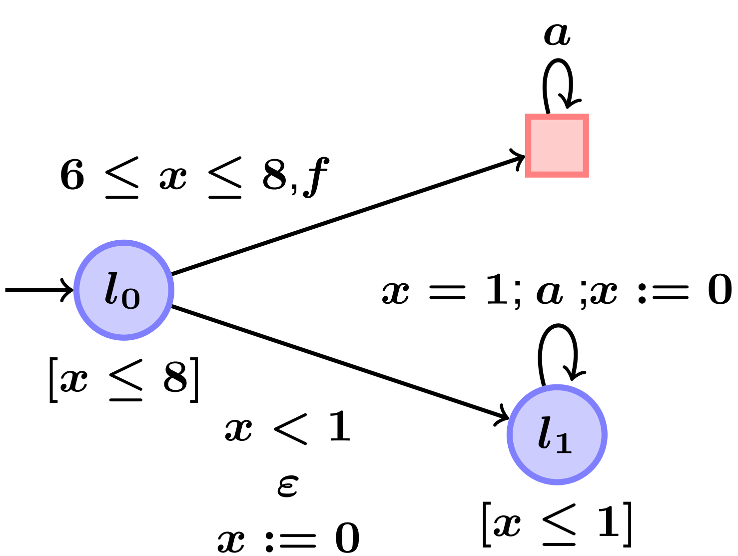

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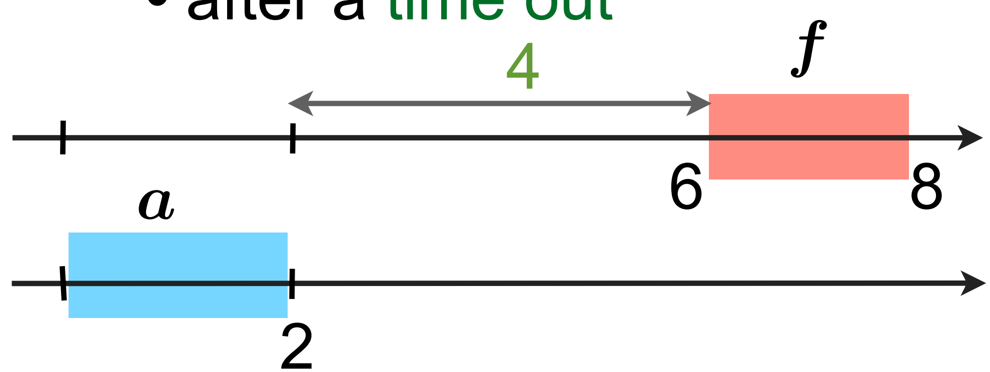

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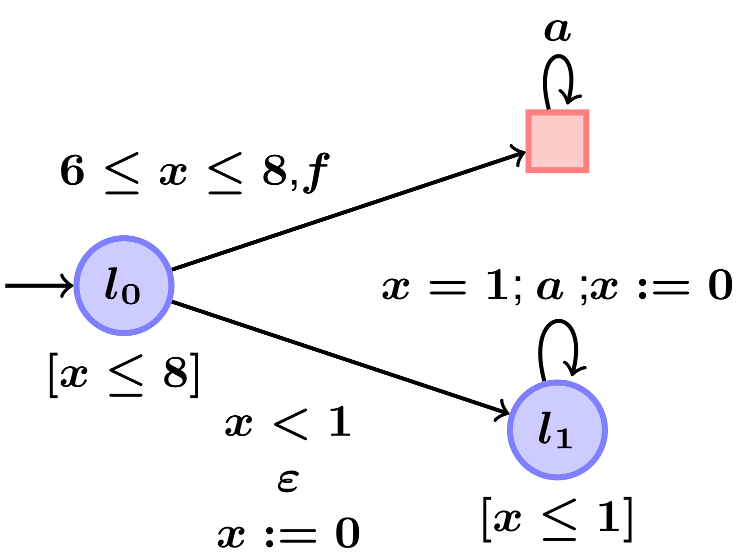

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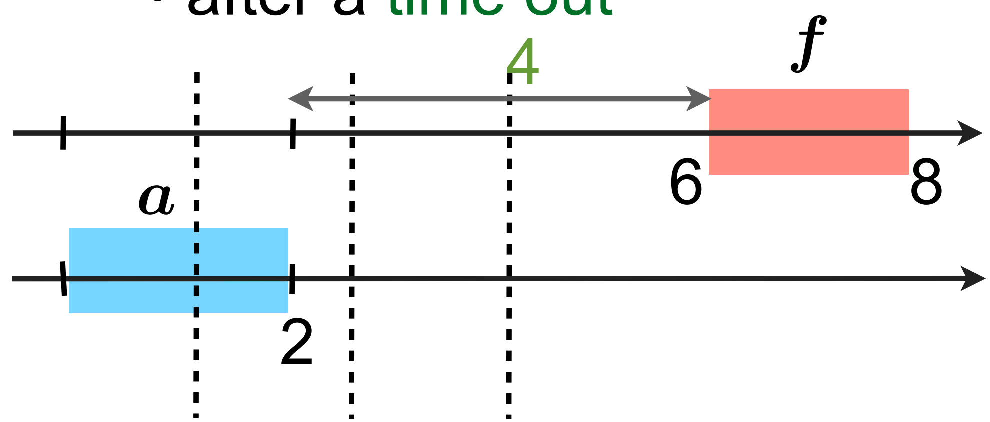

Sampling Predictability

Sampling rate $\alpha \in \mathbb{Q}$

(k, α) – predictor: mapping P that satisfies:

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Checking sampling predictability:

impose $y := 0$ at multiple of α

Sampling predictability: there exists a sampling predictor

Conclusion

- Simple definition of predictability
- Complexity of predictability for TA
  - PSPACE-complete
- Implementability
  - sampling predictability

- Uniform definition for predictability and diagnosability
- Dynamic observers