Synthesis Of Optimal-Cost Dynamic Observers for Fault Diagnosis of Discrete-Event Systems

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Outline of the talk

- Fault Diagnosis for Finite State Systems
  - Fault Diagnosis Problem
  - Sensor Minimization Problems
- Fault Diagnosis with Dynamic Observers
  - Dynamic Observers
  - Cost of an Observer
  - Synthesis of Optimal Dynamic Observers

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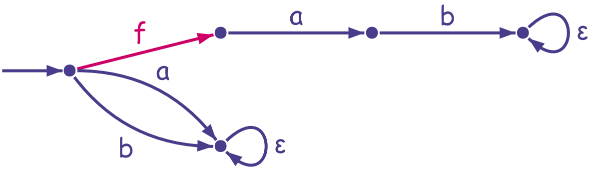
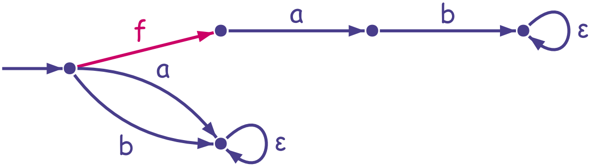
Fault Diagnosis



- A finite automaton $\mathcal{A}$ over $\Sigma^{\varepsilon,f} = \Sigma \cup \{\varepsilon, f\}$
- f is the fault action, Σ is the set of observable events

- k-faulty run contain f followed by more than k actions **$\text{Faulty}_{\geq k}(\mathcal{A})$**
- Non faulty run: contains no f **$\text{NonFaulty}(\mathcal{A})$**

Fault Diagnosis



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> Aim: observe Σ^* sequences and detect k-faulty runs

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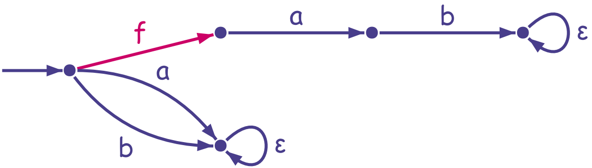

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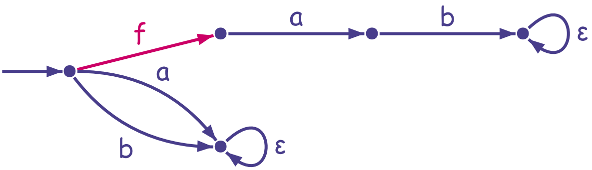
Role of an observer:

- never raise an alarm on non-faulty runs
- must raise an alarm on k-faulty runs

Diagnosis Problem



$tr(\rho)$ = trace of the run ρ (it is a word in $(\Sigma \cup \{f\})^*$)
$\pi_{/\Sigma}(tr(\rho))$ = projection of the trace of the run on observable events

Definition (k-diagnoser)

A mapping $D : \Sigma^* \longrightarrow \{0, 1\}$ is a k-diagnoser for A if:

- for each run $\rho \in$ **NonFaulty**(A), $D(\pi_{/\Sigma}(tr(\rho))) = 0$;
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(Σ, k)-Diagnosability Problem

Given A, Σ, $k \in \mathbb{N}$, is there a k-diagnoser for A?

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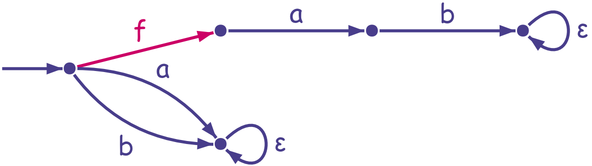

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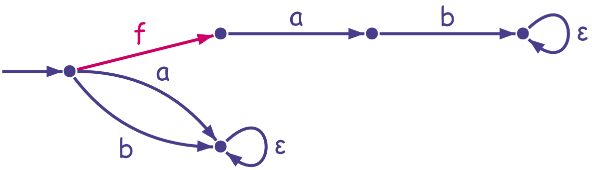

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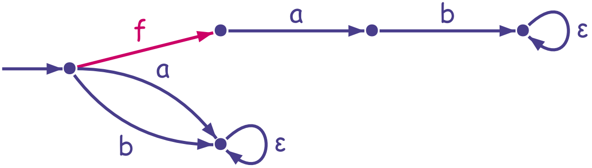

- $\Sigma = \{a, b\}$. Is the plant 1, 2-diagnosable?
- $\Sigma = \{b\}$. Is the plant 1, 2, k-diagnosable?

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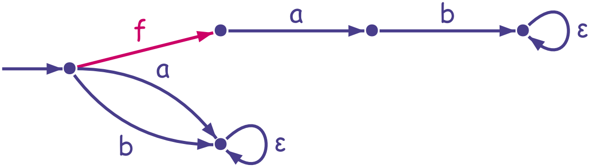

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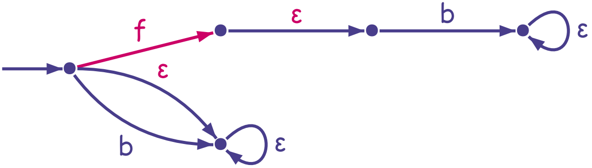

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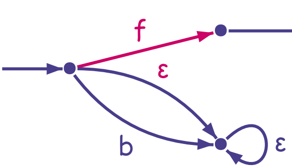

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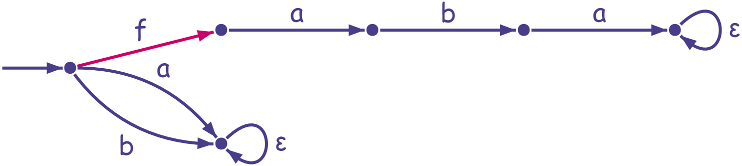

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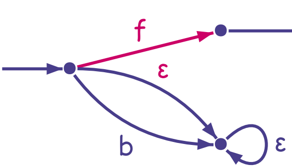

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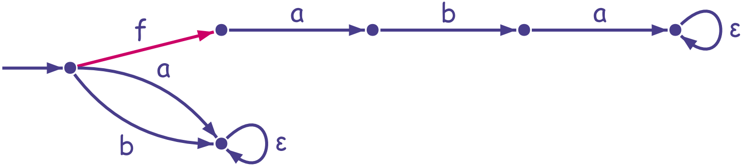

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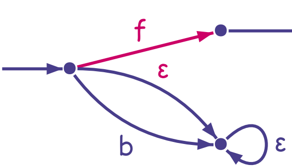

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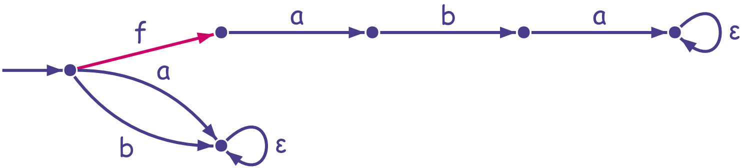

- $\Sigma = \{a\}$. Is the plant 1, 2-diagnosable? 3-diagnosable

Results for the Diagnosis Problem

[Sampath et al., IEEE TAC 1995, Jiang et al., IEEE TAC 2001]

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Diagnosis Problems

- (A) Is A Σ-diagnosable ?
- (B) If "yes" to (A), compute the minimum k and
- (C) Compute a witness diagnoser.

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- (A) is in PTIME
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A witness diagnoser is an automaton with at most $2^{O(A)}$ states

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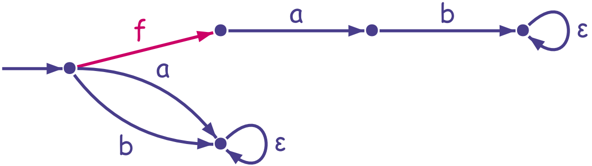

- Aim: find $\Sigma_o \subseteq \Sigma$ s.t. A is Σ_o-diagnosable and minimize $|\Sigma_o|$
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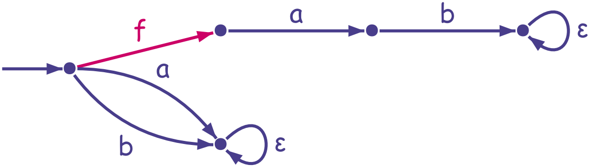

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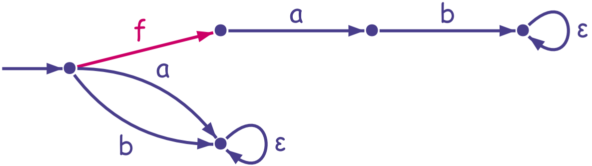

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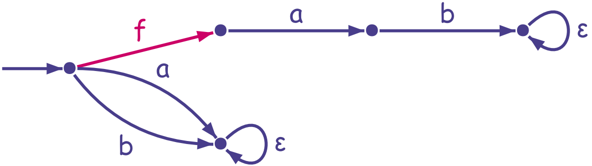

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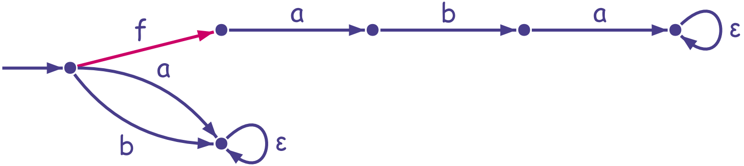

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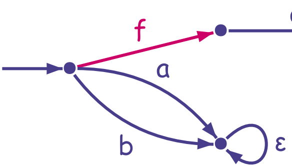

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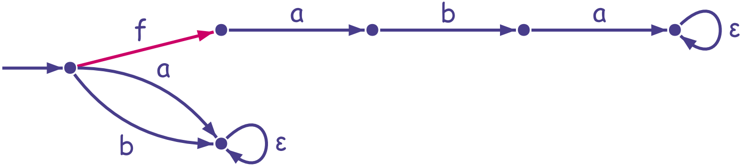

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Results for Sensor Minimization Problems

(Static Minimization Problem)

Input: A, $n \in \mathbb{N}^*$ s.t. $n \leq |\Sigma|$.
Problem: Is there any $\Sigma_o \subseteq \Sigma$, $|\Sigma_o| \leq n$, s.t. A is Σ_o-diagnosable ?

Results:

- Static Minimization Problem is NP-complete
  [Yoo et al., *ACC 2001*, C. Tripakis Altisen, *ACSD 2007*]
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Previous work: Static Observers

In this paper: Dynamic Observers

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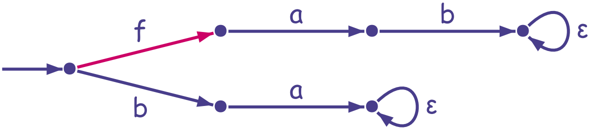
Why Dynamic Observations ?



- Static observer: fixed set of observable events
- Static observation: $|\Sigma_o| \geq 2$, $\Sigma_o = \{a, b\}$; $(\{a, b\}, 1)$-diagnosable
- Dynamic observer: choose dynamically the set of observable events

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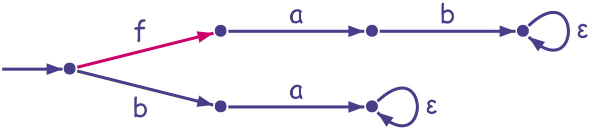

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Assume you can choose Σ_o dynamically:

> Is the plant diagnosable observing only one event at a time ?

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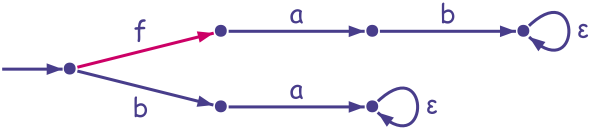

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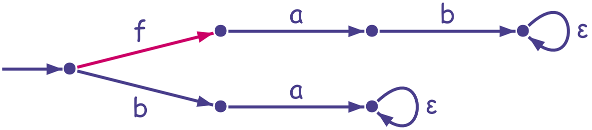

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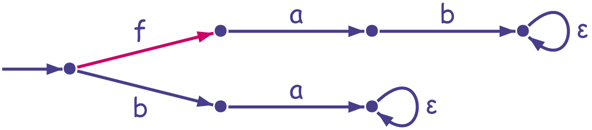

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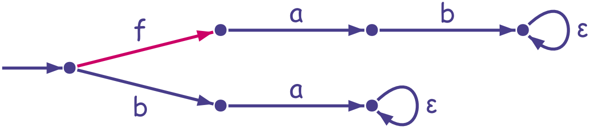

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The plant is dynamically 2-diagnosable

Dynamic Observers

Definition (Dynamic Observer)

A dynamic observer Obs is a complete and deterministic labeled automaton $(S, s_0, \Sigma, \delta, L)$ s.t. $\forall s \in S$, $\forall a \in \Sigma$, if $a \notin L(s)$ then $\delta(s, a) = s$.

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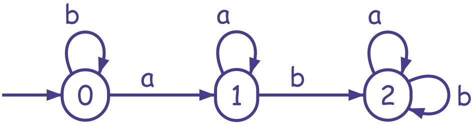

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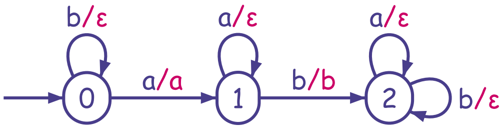

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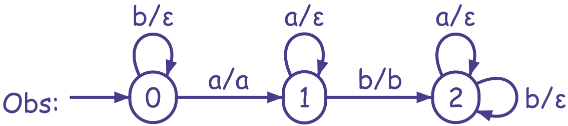
Dynamic Observer and Diagnosability

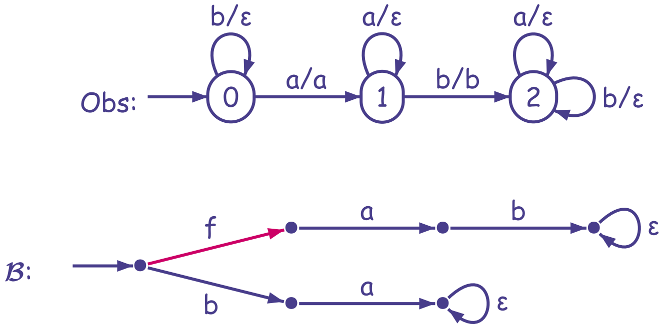


$\mathcal{B}$ is $(\text{Obs}, 2)$-diagnosable.

Define $D(a.b.\rho) = 1$ and $D(\rho) = 0$ otherwise. D is 2-diagnoser.

Obs observes only one event in each state.

Dynamic Observer and Diagnosability



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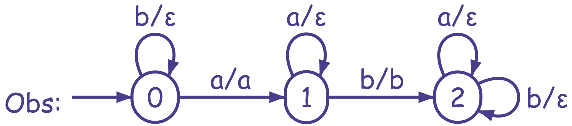
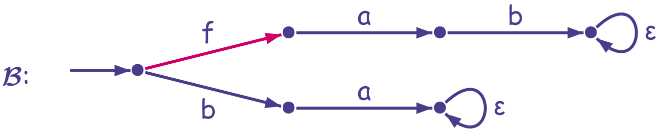
Obs observes only one event in each state.

Dynamic Observer and Diagnosability





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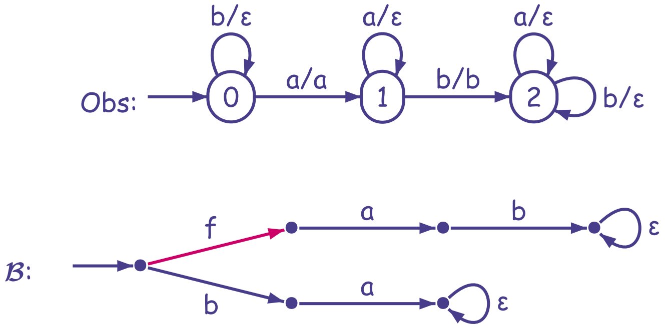

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Checking Obs-diagnosability

[C. Tripakis Altisen, ACSD 2007]

(Obs, k)-diagnosability: A is (Obs, k)-diagnosable if there is an (Obs, k)-diagnoser for A

Obs-diagnosability: A is Obs-diagnosable if $\exists k \in \mathbb{N}$ s.t. A is (Obs, k)-diagnosable

Finite Obs-diagnosability Problem

Input: A, a finite state observer Obs.
Problem: Is A Obs-diagnosable ?

To check Obs-diagnosability, build a product $A \otimes Obs$:

- initial state: (q_0, s_0)
- $(q, s) \xrightarrow{\beta} (q', s')$ iff $q \xrightarrow{\lambda} q'$, $s \xrightarrow{\lambda/\beta} s'$ for $\lambda \in \Sigma$,
- $(q, s) \xrightarrow{\lambda} (q', s)$ iff $q \xrightarrow{\lambda} q'$, and $\lambda \in \{\varepsilon, f\}$

A is Obs-diagnosable iff $A \otimes Obs$ is Σ-diagnosable

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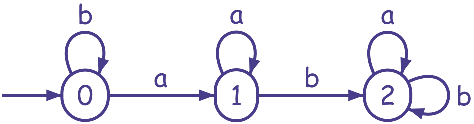
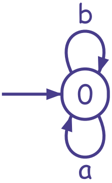
Comparing Dynamic Observers

O_1:



$L(0) = \{a\}$ $L(1) = \{b\}$ $L(2) = \emptyset$

O_2:



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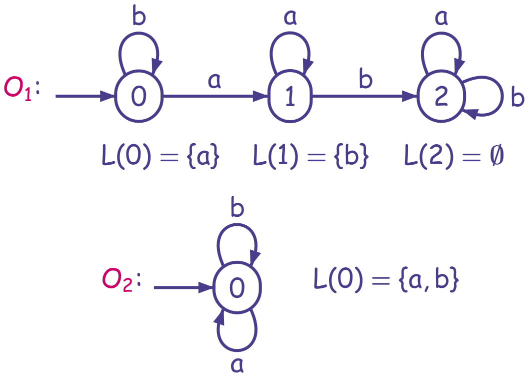
- O_1 is less expensive than O_2

New Problem: compute an optimal observer

Comparing Dynamic Observers



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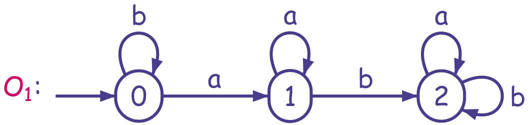

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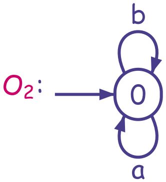

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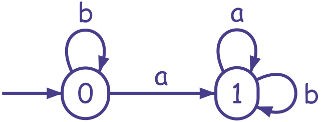
Cost of an Observer

Cost of a run: average number of events observed along a run

Let $Obs = (S, s_0, \Sigma, \delta, L)$



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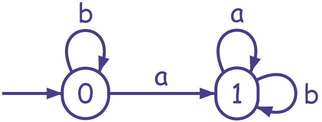

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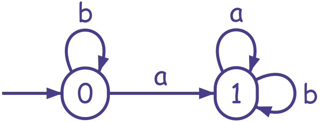

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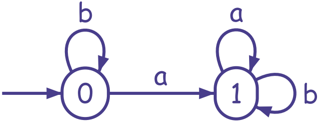

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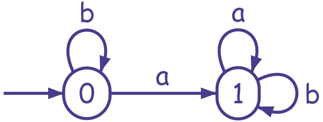

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Computing the Cost of a Given Observer

$$\rho = q_0 \xrightarrow{a_1} q_1 \cdots q_{n-1} \xrightarrow{a_n} q_n \text{ a run of the plant } A$$

Let *Obs* be an observer and $w_i = \pi_{/\Sigma}(tr(q_0 \cdots q_i))$

$$\mathbf{Cost}_2(\rho, Obs) = \frac{1}{n+1} \cdot \sum_{i=0}^{n} |L(\delta(s_0, w_i)|$$

Maximal Cost of runs of length n:

$$\mathbf{Cost}_2(n, A, Obs) = \max\{\mathbf{Cost}_2(\rho, Obs) \textit{ for } \rho \in Runs^n(A)\}$$

The Cost of the pair (Obs, A) is

$$\mathbf{Cost}_2(A, Obs) = \limsup_{n \to \infty} \mathbf{Cost}_2(n, A, Obs)$$

Theorem

$\mathbf{Cost}_2(A, Obs)$ *can be computed in PTIME.*

How to compute the best or optimal observer ?

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Use Karp's Maximum Mean-Weight Cycle Algorithm

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Bounded Cost Observer

Problem 2: Bounded Cost Observer

Input: A, $k \in \mathbb{N}$ and $c \in \mathbb{N}$.
Problem:

(A). Is there an observer Obs s.t. A is (Obs,k)-diagnosable and $\mathbf{Cost}_2(\text{Obs}) \leq c$?

(B). If the answer to (A) is "yes", compute a witness observer Obs with $\mathbf{Cost}_2(\text{Obs}) \leq c$.

Steps to Solve Problem 2:

- Step 1: Compute the most liberal observer O;
  i.e. obtain a representation of the set of all observers
- Step 2: Compute an optimal cost observer.

Theorem ([C. Tripakis Altisen, ACSD 2007])

There is a finite state most permissive observer for A.

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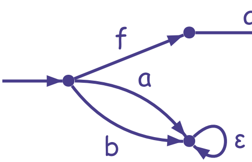
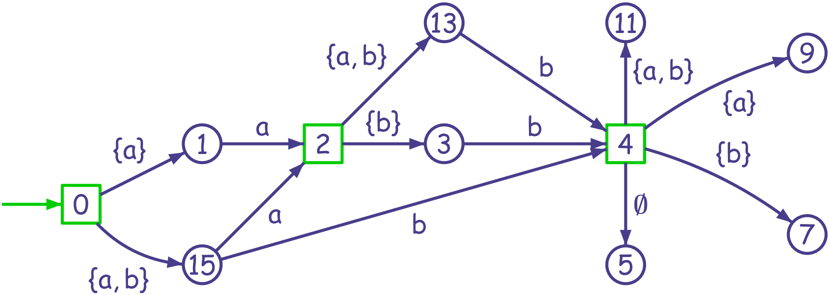
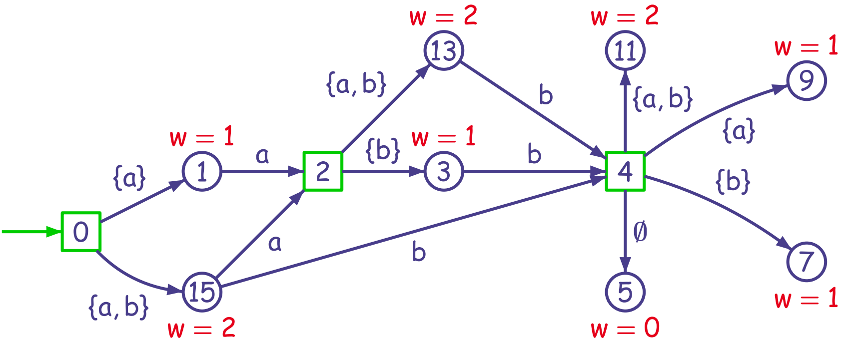
Example: Most Permissive Observer





Optimal Dynamic Observer/Two-Player Game



- Player 1 chooses what to observe: X / Player 2 generates $\rho.l$, $l \in X$
- Player 1 and 2 produce plays
- given a strategy for Player 1 and the moves of Player 2

$$w(\rho) \text{ is the sum of the weights } w_1 w_2 \cdots w_n \text{ of the play } \rho$$
$$Cost_2(\rho, Obs) = w(\rho)/(|\rho| + 1)$$

- Goal for Player 1: minimize $\limsup_{n \to \infty} \{ w(\rho)/(|\rho| + 1) \mid \rho \in \mathit{Runs}^n(A) \}$

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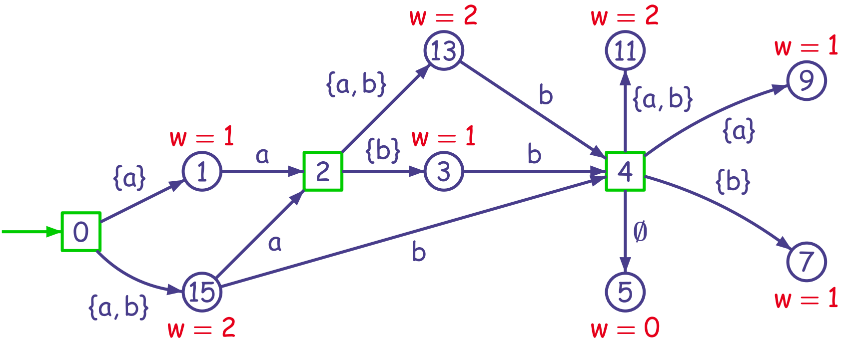

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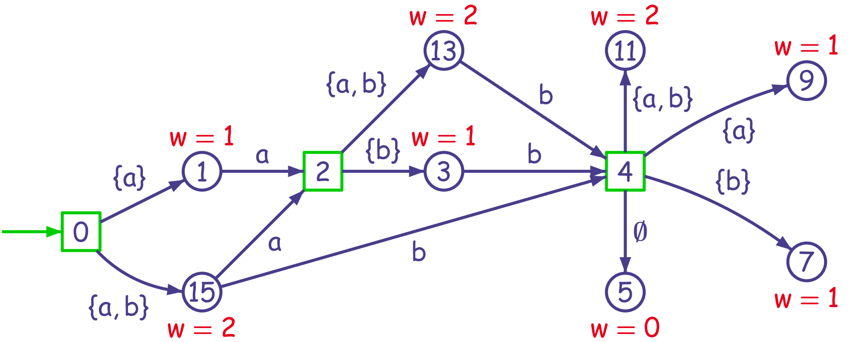

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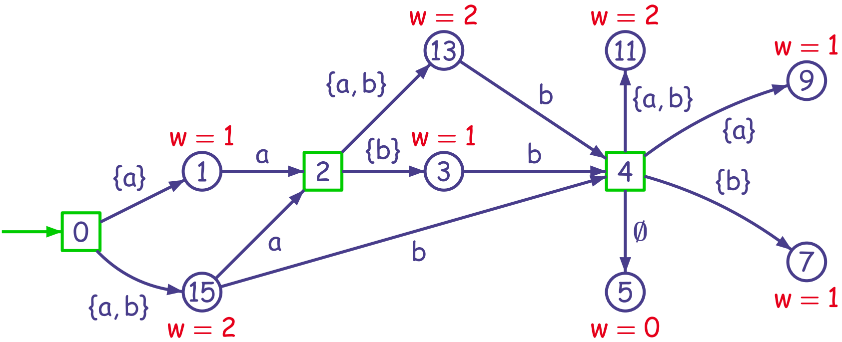

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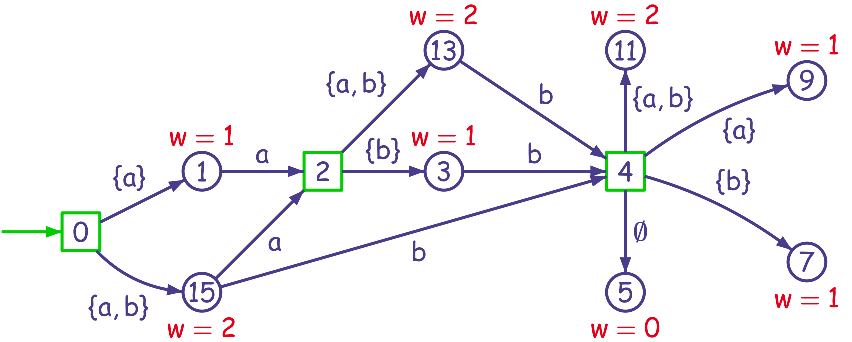

- Player 1 chooses what to observe: X / Player 2 generates $\rho.l$, $l \in X$
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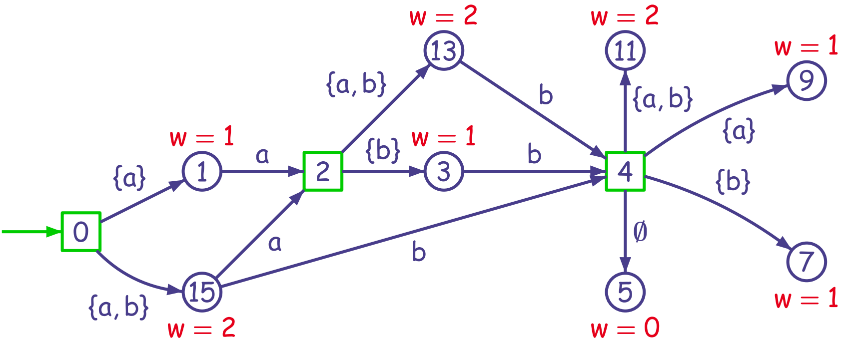

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- Weighted two-player games
- Each state s has a weight w(s)
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  - $\exists v \in \mathbb{Q}$ s.t. each player has a memoryless strategy to ensure $l_0 \leq v$ and $l_1 \geq v$
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- Dynamic observers for bounded diagnosis (k-diagnosability)
- Computation of the most permissive observer
  [C. Tripakis Altisen, ACSD 2007]
- Cost & computation of the cost of a dynamic observers
- Existence of a finite optimal dynamic observer
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- Exact complexity of Problem 2
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References

[C. Tripakis Altisen, 2007] Franck Cassez, Stavros Tripakis, and Karine Altisen.
Sensor minimization problems with static or dynamic observers for fault diagnosis.
Technical Report RI-2007-1, IRCCyN/CNRS, 1 rue de la Noë, BP 92101, 44321 Nantes Cedes, France, January 2007.
Available at `http://www.irccyn.fr/franck`.

[C. Tripakis Altisen, ACSD 2007] Franck Cassez, Stavros Tripakis, and Karine Altisen.
Sensor minimization problems with static or dynamic observers for fault diagnosis.
In *7th International Conference on Application of Concurrency to System Design (ACSD'07)*, Bratislava, Slovak Republic, July 2007. IEEE Computer Society.

[Dasdan et al., IEEE DAC 1999] Ali Dasdan, Sandy S. Irani, and Rajesh K. Gupta.
Efficient algorithms for optimum cycle mean and optimum cost to time ratio problems.
In *Annual ACM IEEE Design Automation Conference*, pages 37–42, New Orleans, Louisiana, United States, 1999. ACM Press New York, NY, USA.
ISBN:1-58133-109-7.

[Debouk et al., DEDS 2004] Rami Debouk, Stéphane Lafortune, and Demosthenis Teneketzis.
On an optimization problem in sensor selection.
Discrete Event Dynamic Systems, 4(12), November 2004.

[Jiang et al., IEEE TAC 2001] Shengbing Jiang, Zhongdong Huang, Vigyan Chandra, and Ratnesh Kumar.
A polynomial algorithm for testing diagnosability of discrete event systems.
IEEE Transactions on Automatic Control, 46(8), August 2001.

[Jiang et al., IEEE TAC 2003] Shengbing Jiang, Ratnesh Kumar, and Humberto E. Garcia.
Optimal sensor selection for discrete event systems with partial observation.
IEEE Transactions on Automatic Control, 48(3):369–381, March 2003.

[Karp, Discrete Math. 1978] Richard M. Karp.
A characterization of the minimum mean cycle in a digraph.
Discrete Mathematics, 23:309–311, 1978.

[R & W, 1987] Peter Ramadge and W. Murray Wonham.
Supervisory control of a class of discrete event processes.
SIAM J. Control Optim., 25(1), January 1987.

References (cont.)

[Sampath et al., IEEE TAC 1995] Meera Sampath, Raja Sengupta, Stephane Lafortune, Kasim Sinnamohideen, and Demosthenis C. Teneketzis.
Diagnosability of discrete event systems.
IEEE Transactions on Automatic Control, 40(9), September 1995.

[Tsitsiklis, 1989] John N. Tsitsiklis.
On the control of discrete event dynamical systems.
Mathematics of Control, Signals and Systems, 2(2), 1989.

[Yoo et al., ACC 2001] Tae-Sic Yoo and Stéphane Lafortune.
On the computational complexity of some problems arising in partially-observed discrete event systems.
In *American Control Conference (ACC'01)*, 2001.
Arlington, VA.

[Zwick & Paterson, TCS 1996] Uri Zwick and Michael S. Paterson.
The complexity of mean payoff games on graphs.
Theoretical Computer Science, 158(1-2):343-359, 1996.