A point of correction to "The petit topos of globular sets"

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March 2, 2022

While reviewing a book, I had occasion to look back at my paper [3]. At the end of Section 1 on page 301 there is a paragraph beginning with the definition of *discrete fibres* for an ω-functor $f : A \to X$. Then $D(\mathrm{omcat}/X)$ is the full subcategory of omcat$/X$ whose objects are the ω-functors into X with discrete fibres. This is all fine but rather irrelevant.

The strange mistake occurs at the bottom of page 307 in Section 4. Here X is now a globular set and $\Psi' : \mathrm{glob}/X \to \mathrm{omcat}/\Psi X$ is induced by the left adjoint $\Psi : \mathrm{glob} \to \mathrm{omcat}$ to the forgetful functor Φ. The full image is stated to be $D(\mathrm{omcat}/\Psi X)$: this is false.

The paragraph on page 301 should have been about unique lifting of factorizations (ulf), not about discrete fibres. An ω-functor $f : A \to X$ is *ulf* when, for all $c \in A_n$ and $x, y \in X_n$ such that $f_n(c) = y \#_k x$, there exist unique $a, b \in A_n$ such that $f_n(a) = x, f_n(b) = y$ and $c = b \#_k a$. For functors between categories and for 2-functors, this terminology was used in [2]. In [1] the term ufl was used instead of ulf in the case of functors. It is very easy to see that ulf ω-functors are stable under pullback in omcat. Also, if $f = g \circ u$ with both f and g ulf then u is ulf. Write $ulf(\mathrm{omcat}/X)$ for the full subcategory of omcat$/X$ whose objects are the ulf ω-functors into X.

Now on page 307, replace the false statement by: the full image of $\Psi' : \mathrm{glob}/X \to \mathrm{omcat}/\Psi X$ is $ulf(\mathrm{omcat}/\Psi X)$. Then on page 308, replace the pseudo-functor $D(\mathrm{omcat}/-)$ throughout by the pseudo-functor $ulf(\mathrm{omcat}/-)$.

I detect no further corrections necessary.

References

[1] Marta Bunge and Susan Niefield, Exponentiability and single universes, *J. Pure Appl. Algebra* **148** (2000) 217–250.

[2] Ross Street, Categorical structures, *Handbook of Algebra Volume 1* (editor M. Hazewinkel; Elsevier Science, Amsterdam 1996; ISBN 0 444 82212 7) 529–577.

[3] Ross Street, The petit topos of globular sets, *J. Pure Appl. Algebra* **154** (2000) 299–315.