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Corrigendum

Corrigendum to "Combinatorial categorical equivalences of Dold–Kan type" [J. Pure Appl. Algebra 219 (10) (2015) 4343–4367]

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ABSTRACT

We make several corrections and improvements to the published paper "Combinatorial categorical equivalences of Dold–Kan type", mostly relating to the standing assumptions of the paper. In particular we have had to add one new assumption, but have been able to remove another.

Since the publication of the paper [2] we have become aware of various corrections which are needed and improvements which are possible. The main result of the paper relied on six assumptions, numbered 2.1 to 2.6. It turns out that Assumption 2.6 is a consequence of the other axioms. On the other hand, there is a gap in the proof of Proposition 6.5 which we have been able to fix only by adding a new assumption, which does still hold in all of the examples considered in the paper. We also take this opportunity to correct several smaller errors.

The new assumption will hold if the category $\mathscr{P}$ to which it applies underlies a suitable locally ordered 2-category.

We have placed on the arXiv a corrected version [3] of the paper.

We are grateful to Clemens Berger and Richard Garner for discussions related to some of these issues.

1. Badly worded assumption

Our Assumption 2.1 was badly worded, so that a reader could easily have failed to grasp its intended meaning. A better wording would be:

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Assumption 2.1. Every morphism $f \in \mathscr{P}$ factors as $f = n \circ r \circ m^*$ for $m, n \in \mathscr{M}$ and $r \in \mathscr{R}$, and these m, n, and r are unique up to isomorphism.

The issue is that it is n, r, and m which are unique up to isomorphism, not just n, r, and m^*.

2. Unnecessary assumption

Assumption 2.6 of the paper is in fact a consequence of the remaining assumptions, as can be seen in the following result, which appears as Proposition 2.7 of the updated arXiv version [3].

Proposition A. *If $t \circ s = m \circ r$ with $s, t \in \mathscr{S}$, $r \in \mathscr{R}$, and $m \in \mathscr{M}$, then both s and t are in $\mathscr{R}$.*

Proof. First note that, by uniqueness in Assumption 2.1, if $x \circ n^* = m' \circ r'$ in obvious notation, then n^* is invertible. Now, with s, t, m, r as in the Proposition, we can put $s = r_1 \circ m_1^*$ and $t = r_2 \circ m_2^*$. Then $r_2 \circ m_2^* \circ r_1 \circ m_1^* = t \circ s = m \circ r$. So m_1^* is invertible and we conclude that $s \in \mathscr{R}$. Using Assumption 2.1, we have $m_2^* \circ s = m_3 \circ r_3 \circ n_1^*$. Then $r_2 \circ m_3 \circ r_3 \circ n_1^* = m \circ r$ implies n_1^* invertible. So we may suppose $m_2^* \circ s = m_3 \circ r_4$. Then $(m_2 \circ m_3)^* \circ s = m_3^* \circ m_2^* \circ s = m_3^* \circ m_3 \circ r_4 = r_4$. By Assumption 2.3, $m_2 \circ m_3$ is invertible. It follows that m_2 is invertible, so $t \in \mathscr{R}$. □

3. Added assumption

As mentioned above, the proof of Proposition 6.5 of the published paper contains a gap. What is actually needed in the proof is that, for an object $A \in \mathscr{P}$, the $\mathscr{M}$-subobjects $m \colon U \preceq A$ of A can be listed as $m_0 = 1_A, m_1, \ldots, m_n$ in such a way that if $m_i^* \circ m_j \in \mathscr{M}$ then $i \leq j$.

Define a relation R_A on the finite set Sub$A$ by $m R_A n$ if $m^* \circ n \in \mathscr{M}$. This relation is reflexive and antisymmetric but not in general transitive. We now require

Assumption B. For all objects $A \in \mathscr{P}$, the relation R_A is contained in some antisymmetric transitive relation on Sub$A$.

under which assumption the proof of Proposition 6.5 becomes valid. This is called Assumption 2.6 in [3]; it remains valid in all of our examples, thanks to the following result, which appears as Proposition 2.9 in [3].

Proposition C. *Suppose each hom-set of $\mathscr{P}$ is equipped with a reflexive, transitive, antisymmetric relation $\leq$ respected by composition on either side; thus we have a locally posetal 2-category $\mathbb{P}$ with underlying category $\mathscr{P}$. Suppose further that, for all $m \in \mathscr{M}$, m^* is right adjoint to m with identity unit. Then Assumption B holds. Dually, the same is true if instead each m^* is left adjoint to m with identity counit.*

Proof. For m and n in $\mathscr{M}$ with codomain A, let $m \trianglelefteq n$ mean that there exists ℓ in $\mathscr{M}$ with $m\ell \leq n$ in $\mathbb{P}$. We claim that $\trianglelefteq$ is transitive, antisymmetric, and contains the relation R_A of Assumption B.

Suppose $m_1 \trianglelefteq m_2 \trianglelefteq m_3$. Then there are m and n in $\mathscr{M}$ with $m_1 m \leq m_2$ and $m_2 n \leq m_3$. So $m_1 m n \leq m_2 n \leq m_3$ yielding transitivity.

Suppose $m_1 \trianglelefteq m_2 \trianglelefteq m_1$. Then there are m and n in $\mathscr{M}$ with $m_1 m \leq m_2$ and $m_2 n \leq m_1$, and so $m_1 m n \leq m_1$; but m_1 is fully faithful, so $mn \leq 1$. This gives a descending chain

$$\cdots \leq (mn)^3 \leq (mn)^2 \leq (mn) \leq 1$$

in $\mathscr{P}(A, A)$. All terms of the chain are in the finite set Sub(A) so they cannot be distinct. So $(mn)^a = (mn)^b$ for some natural numbers $a > b$. Since mn is a monomorphism, $(mn)^{a-b} = 1$; so m is a retraction and a monomorphism, hence invertible. Thus $m_1 = m_2$, proving $\trianglelefteq$ antisymmetric.

If mR_An then $m^* \circ n$ is equal to some $\ell \in \mathscr{M}$. By the adjointness $m \dashv m^*$, it follows that $m\ell \leq n$, and so $m \trianglelefteq n$. This proves that R_A is contained in $\trianglelefteq$. □

The 2-category structure for Example 7.3 is described in the paper; that for Example 7.2 is well-known and arises from the fact that $\Delta_{\perp,\top}$ is a full subcategory of **Cat**; while that for Example 7.1 is also well-known, and can be found for example in [1].

Remark D. In fact, in each of the examples of the paper, the existence of an adjoint as in Proposition C suffices to *characterize* the classes $\mathscr{M}$, $\mathscr{R}$, and $\mathscr{M}^*$, as well as the correspondence $m \mapsto m^*$. It is striking that, although the 2-category structure plays no role in the statement of the theorem, it does determine a choice of these classes of maps, as well as simplifying the verification of the assumptions.

4. Other minor corrections

Richard Garner pointed out to us that the hypotheses of Proposition 2.7 were inadequate. The correct statement is:

Proposition 2.7. *Assume that the pullback of each morphism in $\mathscr{M}$ along any morphism in $\mathscr{R}$ exists and is in $\mathscr{M}$. Assume wide pullbacks of families of morphisms in $\mathscr{M}$ exist, have projections in $\mathscr{M}$, and become wide pushouts under $m \mapsto m^*$. Then Assumptions 2.1 and 2.2 hold.*

In Example 7.3, there was a missing "op": in fact we obtain an equivalence $[\mathbb{I}, \mathscr{X}] \simeq [\Delta_{\text{inj}}^{\text{op}}, \mathscr{X}]$.

On the third and fourth lines after equation (1.2) in the introduction, the correct statement is that "cubical abelian groups are equivalent to semi-simplicial abelian groups".

References

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