Example 11 of Day-Street "Monoidal bicategories..."

We will take the antipode axiom at the bottom of page 142 in the form of a $\mathcal{V}$-functor $S : \mathcal{H} \to \mathcal{H}^{op}$ together with a $\mathcal{V}$-natural isomorphism

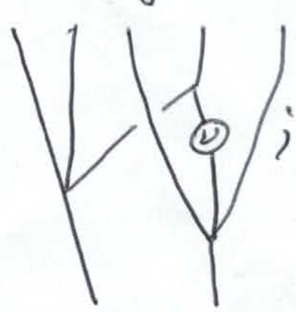
$$(*) \quad \mathcal{H}(B,C) \otimes \mathcal{H}(A,SB) \cong \mathcal{H}(B,C) \otimes \mathcal{H}(A,C).$$

<u>Proposition</u> <u>Each Hopf monoid</u> H <u>in a braided monoidal category</u> $\mathcal{V}$ <u>gives a one-object example of a Hopf</u> $\mathcal{V}$-<u>algebroid</u>.

<u>Proof</u> Here $\mathcal{H}$ has one object A and $\mathcal{H}(A,A) = H$ so that $(*)$ becomes an isomorphism

$$H \otimes H \cong H \otimes H$$

for which we take the fusion map . To see that it gives a $(*)$ which is $\mathcal{V}$-natural, we must see what functoriality of $(*)$ in A, B, C means on the two sides. Both sides become left H-, right $H \otimes H$-modules. The left $H \otimes H$ has action given by

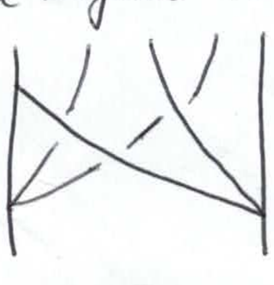


; i.e.

$$H^{\otimes 5} \xrightarrow{H^{\otimes 3} \otimes \delta \otimes H} H^{\otimes 6} \xrightarrow{H^{\otimes 4} \otimes \nu \otimes H} H^{\otimes 6}$$

$$\xrightarrow{H^{\otimes 2} \otimes c_{H,H} \otimes H^{\otimes 2}} H^{\otimes 6} \xrightarrow{\mu_3 \otimes \mu_3} H^{\otimes 2}.$$

The right $H \otimes H$ has action given by

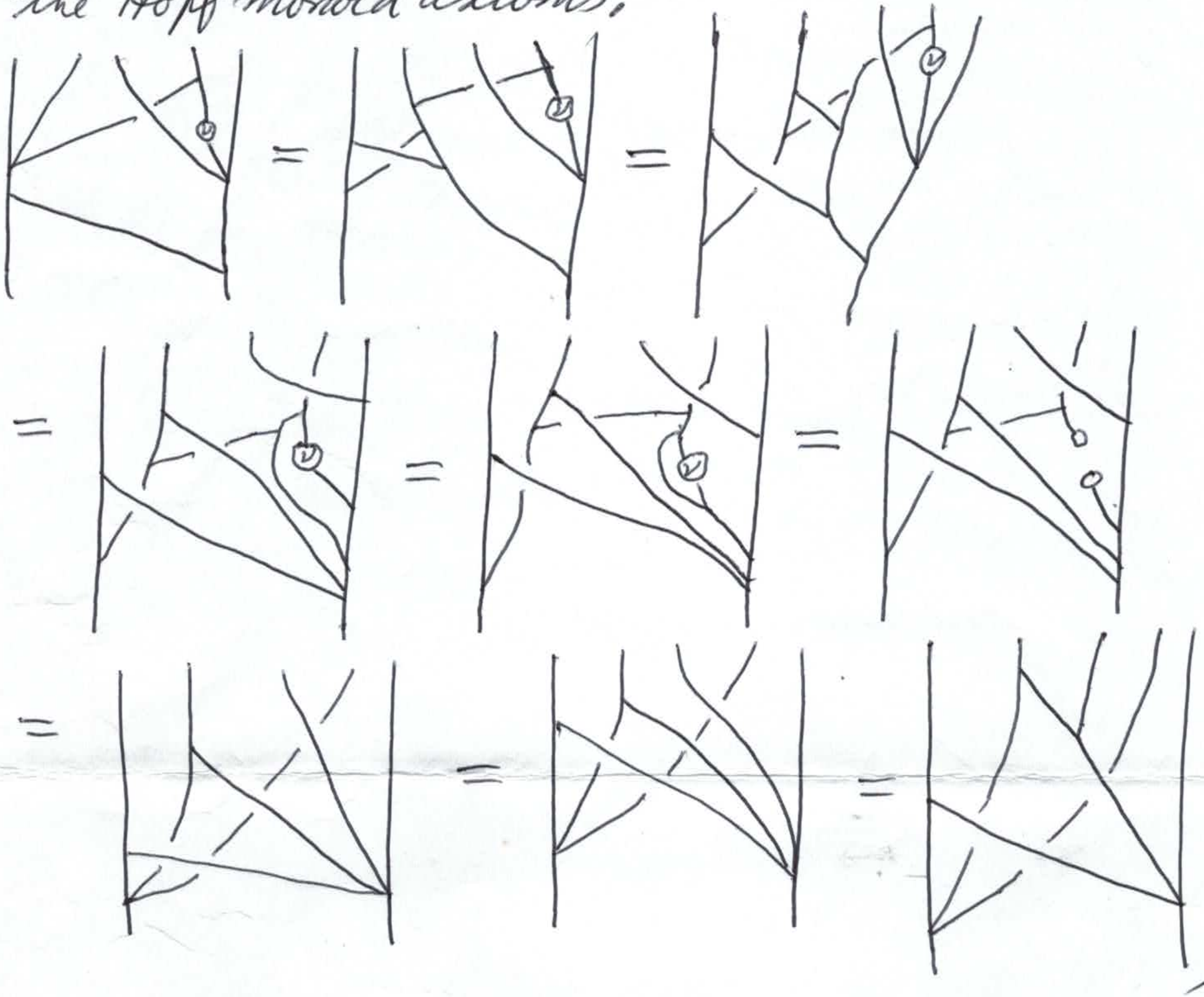


; i.e.

$$H^{\otimes 5} \xrightarrow{\delta \otimes H^{\otimes 4}} H^{\otimes 6} \xrightarrow{H \otimes c_{H,H} \otimes c_{H,H} \otimes H} H^{\otimes 6}$$

$$\xrightarrow{H^{\otimes 2} \otimes c_{H,H} \otimes H^{\otimes 2}} H^{\otimes 6} \xrightarrow{\mu_3 \otimes \mu_3} H^{\otimes 2}.$$

The proof is completed by a string calculation using the Hopf monoid axioms:



as required. □