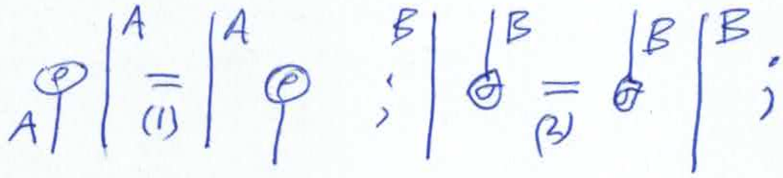
Duality and well pointedness Sept 2017

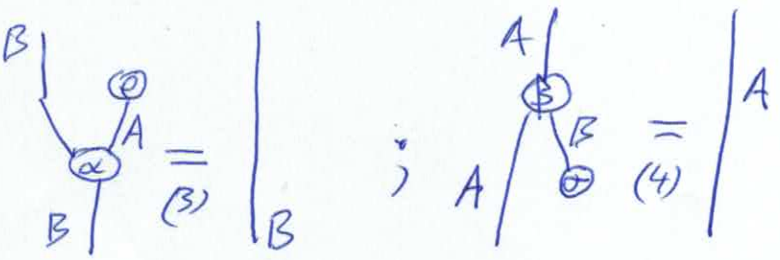
V any monoidal category

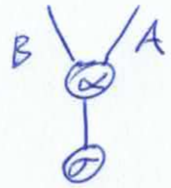
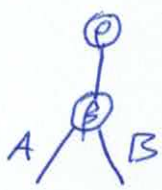
$\rho : I \longrightarrow A$ is a well-pointed object

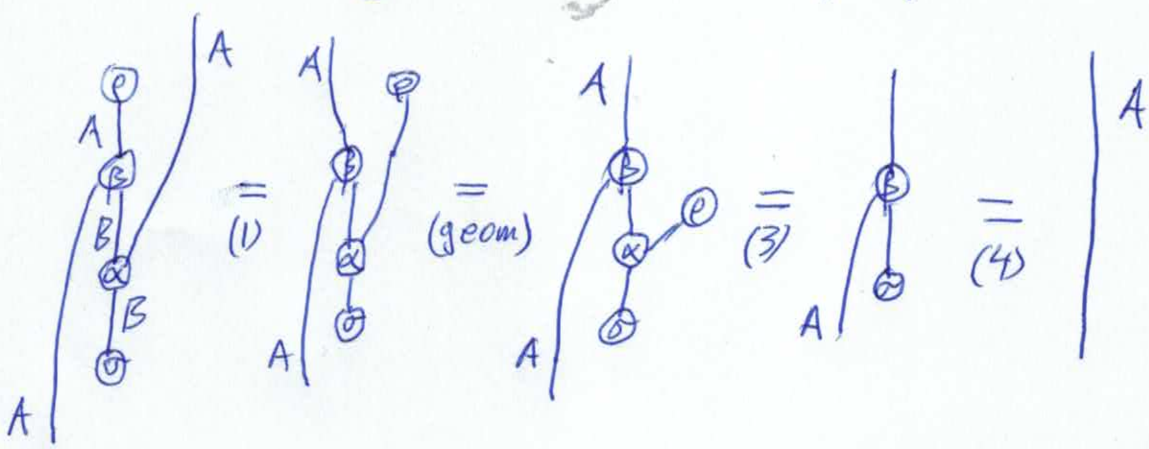
$\sigma : B \longrightarrow I$ is a well-copointed object

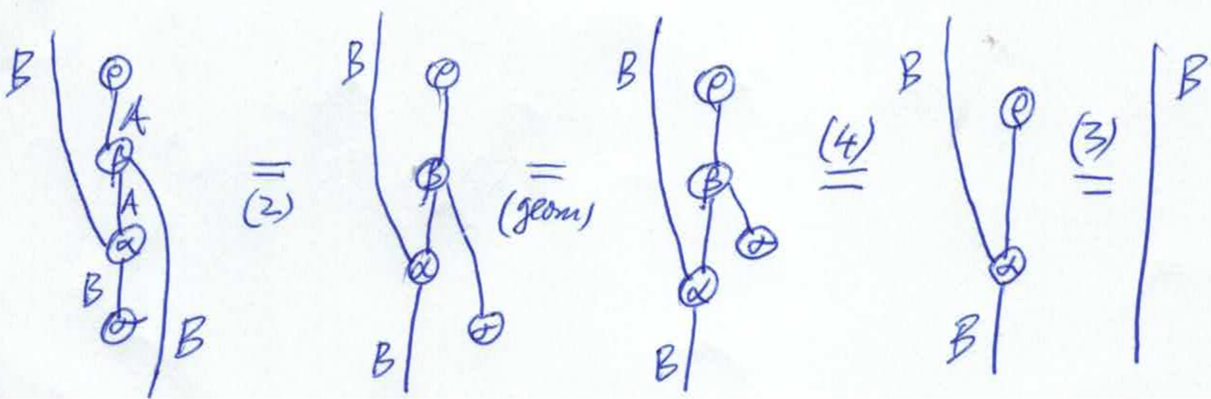
If $B \otimes \rho$ is a coretraction and $A \otimes \sigma$ is a retraction then $B \dashv A$ and ρ, σ are mates.

Proof 

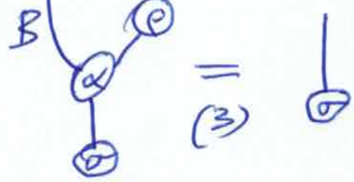


Put $\varepsilon =$  ; $\eta =$ 





The mate of $I \xrightarrow{\rho} A$ is $B \xrightarrow{B\otimes\rho} B\otimes A \xrightarrow{\varepsilon} I$; but this is σ since



$\cdot$ $\square$

<u>Corollary</u> <u>For</u> $X \in \mathcal{V}$, <u>the following are equivalent</u>:

(i) $\rho\otimes X : X \longrightarrow A\otimes X$ is invertible;

(ii) $\rho\otimes X : X \longrightarrow A\otimes X$ is a coretraction;

(iii) $\sigma\otimes X : B\otimes X \longrightarrow X$ is invertible;

(iv) $\sigma\otimes X : B\otimes X \longrightarrow X$ is a retraction.

<u>Remark</u> This was inspired by Theorems 6 and 7 of Barr's paper in Tbilisi Math J 10(3) 2017.