population numbers, and the various types of equilibrium such as stable foci and limit cycles are treated in detail. Chapter Four, on biogeography, gives a mainly algebraic treatment of the problem on how many species a given area can accommodate under processes of immigration and emigration, and applies this to the optimum design of wildlife reserves.

Chapter Five is concerned with the movement of metabolites through an organism, modelled as a system of linked compartments. After some discussion of the general case, the tractable system of the distribution of a drug between two compartments (blood and tissue) is dealt with in more detail.

The next four chapters, Six to Nine, cover almost half the book and treat the flow of blood in elastic-bounded vessels such as arteries and heart chambers. Here is a rich field of applications of classical applied mathematics, with hydrodynamics and elasticity combined. There are too many investigations to list in a short review, but throughout these chapters runs one continuing theme: to show how physical properties such as the elasticity of artery walls, heart valves, etc., can be determined from measurements that can be taken without surgical intervention, and thus provide a diagnosis of abnormalities.

The last of these chapters, number Nine, on analysis and application of heart valve vibration, differs from the rest of the book in that it is largely a summary of results developed elsewhere. However, detailed references are supplied.

The short Chapter Ten describes some of the medical devices by which the measurements referred to in the previous chapters can be made. This is followed by a glossary of medical terms, necessary for most mathematicians and adequate for understanding the later sections except some of Chapter Nine. Finally, there is a good list of references and an index.

At the end of each chapter is a selection of problems, some being straightforward applications of the material in the text and some being extensions of that material. This is a book that can be used for a lecture course or for independent study. It can be recommended to anyone wishing to become acquainted with further interesting applications of mathematics.

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LIE GROUPOIDS AND LIE ALGEBROIDS IN DIFFERENTIAL GEOMETRY

Kirill Mackenzie

(London Mathematical Society Lecture Note Series **124**, Cambridge University Press, 1987)

Reviewed by Ross Street

The work on smooth symmetries begun by Sophus Lie and continued by Elie Cartan is only partly understood through the study of Lie groups and Lie algebras. Various structures have been proposed as vehicles for this subject—the most documented being smooth principal algebra bundles. This is the first book on a very attractive alternative.

The algebraic concept of groupoid goes back to Heinrich Brandt (1926) and provides a natural generalisation of the concept of group (despite the negative psychological connotations of the suffix). Twenty years later one could have said that a groupoid was precisely a (small) category in which all morphisms were invertible. Yet little was made of this special class of categories until the next decade when Charles Ehresmann proposed them as fundamental in differential geometry.

The case for the importance of groupoids in mathematics is overwhelming. Just as the automorphisms of a structure form a group, the isomorphisms in any category form a groupoid; this provides a wealth of examples. The fundamental group of a space X depends on the arbitrary choice of a point; whereas the merits of the fundamental groupoid $\pi(X)$, whose objects are the points of X and whose morphisms are homotopy classes of paths, have been convincingly illustrated by Ronnie Brown [2, 3]. In fact, $\pi(X)$ is a topological groupoid: there are compatible topologies on the sets of objects and morphisms. Topological groupoids have recently been shown to play a dominant role in the structure of toposes.

[At this point I would like to recommend the excellent reviews of the present book by Kock [4] and Kumpera [5]. Mackenzie's assurance in the Introduction is that "... we make no actual use of category theory". Readers fancying a review in the same spirit should especially consult those alternatives.]

In 1951 Ehresmann defined differential groupoids and, soon after, differential categories. I believe this was the beginning of Ehresmann's interest in categories. His motivation was vastly different from that of Eilenberg–Mac Lane for whom categories were generally large collections of structures. The idea that *functions* could be points of a space was adequately dealt with by modern mathematics; yet the ancient idea that *structures* (such as spaces) could also be points of a space was not. Ehresmann's substantial advance was the recognition that categories themselves were fundamental algebraic structures in their own right; indeed, he went on to develop the theory of categories within other categories (topological categories are categories within the category of topological spaces). For example, if p: $E \to B$ is a map in a particularly nice category $\mathbf{S}$, the frame groupoid $\Pi(E)$ within $\mathbf{S}$ has the elements of B as objects and isomorphisms $E_b \to E_c$ between the fibres as morphisms. It took at least another decade even for category theorists to generally recognise the significance of Ehresmann's contribution.

Ehresmann's student Jean Pradines began the serious study of the infinitesimal aspects of differential groupoids. In particular, he is responsible for the concept of (and inevitable term) Lie algebroid on a base manifold B. This amounts to a vector bundle A over B together with a bundle map $A \to TB$ into the tangent bundle on B and a bracket operation on the global sections ΓA of A subject to conditions. Each differential groupoid with base manifold B of objects has an associated Lie algebroid with base B. The Atiyah sequence plays this role in the principal bundle approach. All that Pradines published of this work was four short announcements [6, 7, 8, 9], the last of which included the tantalising claim that Lie's Third Theorem generalised: each transitive Lie algebroid is associated to a differential groupoid!

Mackenzie in the mid-1970s, as a postgraduate student at Monash University supervised by Juraj Virsik, began the considerable task of providing full proofs for Pradines' announcements. The Third Theorem resisted. Motivated by van Est's cohomological techniques [10], Mackenzie reduced the problem to the non-vanishing of a certain quantity. Meanwhile, Almeida in Brazil was also working on Pradines' notes. Much of interest came from these studies, but the big surprise had to wait until 1985: Almeida and Molino [1] produced Lie algebroids not associated with Lie groupoids. This gave "an entirely new insight into the subject" [p. 259]. Mackenzie's quantity was then seen as the 2-cocycle representing the obstruction to integrability of the Lie algebroid.

With these developments, a detailed account in book form was certainly in order. Mackenzie's book does much more than this. It is a scholarly, readable treatment of the basic theory of topological groupoids, differential groupoids, and Lie algebroids. The subject which gives the book its depth is the cohomology of Lie algebroids in Chapter IV including a large amount of the author's original, previously unpublished work. Much of the connection theory is done here, without reference back to the Lie groupoid. An Appendix on the "traditional approach" using principle bundles and Atiyah sequences is provided for completion.

This book should certainly be on the shelves of anyone serious about differential geometry. I believe it is consistent with the geometric spirit sought by Ehresmann in his foundational work. It is the first book in an intriguing field and will no doubt stimulate future developments to which I look forward.

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COMBINATORICS OF EXPERIMENTAL DESIGN

Anne Penfold Street and Deborah J. Street

(Oxford University Press, 1987)

Reviewed by Alan Rahilly

It is desirable at the upper levels of an undergraduate mathematics curriculum to incorporate courses which reinforce fundamental material dealt with in earlier courses. Design theory, since its development involves the use of basic results in number theory, linear algebra and abstract algebra, is one topic which possesses considerable attraction in this regard. *Combinatorics of Experimental Design* by Street and Street is aimed at an audience of third or fourth year mathematics or statistics students. As its title indicates, its emphasis is on combinatorial aspects of experimental design. However, quite some space is devoted to making clear the statistical relevance of designs. This is particularly evident in Chapter 1, where designs and the linear models are discussed, and in Chapter 8, 9, 14 and 15 which deal with factorial, neighbour and competition designs. The remaining chapters contain a well chosen selection of basic material on designs, with pairwise balanced designs (especially balanced incomplete block designs) figuring prominently.

In the theory of designs existence problems are of central importance. The book under review gives this due recognition. For example, in Chapter 5 one finds an efficient handling of the existence problem for triple systems and, in Chapter 7, the proof that $N(n) \geq 3$ for

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