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Elementary categories, elementary toposes. (English) [Zbl 0762.18001](#)
Oxford Logic Guides. 21. Oxford: Clarendon Press. xiii, 265 p. (1992).

Perhaps the most beautiful achievement of category theory is the discovery by Lawvere-Tierney of the concept of elementary topos. The axioms are so simple for the power they contain. Yet the mathematician or student wanting to learn the subject has had the daunting task of first learning some category theory from, say, *S. Mac Lane's* book [Categories for the working mathematician (1971; [Zbl 0232.18001](#))], and then moving to a book on topos theory, say, *P. T. Johnstone's* [Topos theory (1977; [Zbl 0368.18001](#))]. It is difficult for the beginner to know which bits of category theory can be omitted for the eventual goal. In fact, the axioms on a category, for it to be an elementary topos, can be expressed in terms of the basic notions of terminal object, product, pullback and monomorphism.

The present book is a good introduction to both categories and toposes. It is divided into four parts: Part I: Categories (where “category”, and the relevant concepts within a single category, are defined); Part II: The Category of Categories (which considers interactions between various categories by means of functors and natural transformations); Part III: Toposes (where “(elementary) topos” is defined, and, in the main, work proceeds within a single topos); and Part IV: Some Toposes (where examples are given).

Features are: (a) there are no categorical prerequisites; (b) the category of categories as a foundation for mathematics is discussed; (c) the internal language is introduced soon after the definition of topos and that language is used for developing the theory; (d) synthetic differential geometry and the effective topos are included. In order to keep the book its appealing size (around 270 pages), each of these features brings its limitations. Very little is said about geometric morphisms, I presume, because that would require too much categorical groundwork. No reference is made to 2-categories and bicategories which is where a reader aroused by (b) must look for further reading. To be really convinced by the approach (c), the reader should have some maturity in mathematical logic; and this knowledge should extend to recursive function theory if the effective topos example is to be completely appreciated.

In all, this is a gentle modern introduction to categories and toposes, and I recommend it.

Reviewer: [R.H.Street \(North Ryde\)](#)

MSC:

[18-01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to category theory
[18B25](#) Topoi

Cited in **3** Reviews
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[category](#); [elementary topos](#); [category of categories](#); [synthetic differential geometry](#); [effective topos](#)