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Gordon, Robert (1-TMPL)
G-categories.
Mem. Amer. Math. Soc. **101** (1993), *no.* 482, xvi+129 *pp.*

The literature of algebraic topology and representation theory is sparsely sprinkled with ad hoc results about categories on which a group G acts; here they are called G-categories. Category theory has a lot to say about G-categories because they are special cases of many structures: for example, categories internal to a presheaf topos, G-graded categories, categories fibred over G (or G-indexed categories), categories with homs enriched in a (nonsymmetric) biclosed monoidal category, algebras for a doctrine. Yet, it is precisely this diversity of views that makes it worthwhile to begin developing a systematic theory, as is done in the present memoir. All the peculiarities of G-categories are explained by no single view, and some by none. The author is motivated, it seems, by the prospect of applications to modules over G-graded algebras. There is particular value in having this work done by someone whose specialty is outside category theory.

Limits, representability, adjointness and cotripleability are all adapted to the G-context. The author is interested in the control the stable subcategory has over its ambient G-category. However, the reviewer believes stability considerations entered too strongly into the choice to consider only conical limits (= Kan extensions along the unique functor into the trivial G-set), and that a more versatile notion would have been Kan extension along functors into the G-set G (acting on itself by multiplication); this would have been discovered had a formula for Kan extensions been included.

For the variant of the Yoneda lemma, a special G-category Par(G-Set) is introduced to receive the hom functors of G-categories. It is interesting to observe that this does actually come out of the viewpoint of G-sets as presheaves on G (as a one-object category). The gross internal full subcategory of the category of presheaves on C in the sense of the reviewer [Trans. Amer. Math. Soc. **258** (1980), no. 2, 271–318; MR0558176] was shown to be the category-valued presheaf on C whose value at $u \in C$ is the category of presheaves on C/u; it can be checked that this is equivalent to Par(G-Set) when $C = G$. Indeed, the G-groupoid C/u is used by the author to discuss systems of isomorphisms and stably closed G-categories.

There is a section on "transversals". Trasversaled functors between G-categories arise, for a category theorist, from the view that G-categories are categories fibred over G; they are then the appropriate morphisms (more general than G-functors). Alternatively, they are pseudonatural transformations between category-valued functors on G. Because of the inverses in G, the author is able to prove that, if one of the functors in an adjunction is a G-functor, then the other admits a transversal but is generally not a G-functor.

The memoir is written for algebraists with a basic knowledge of category theory, yet there is much to stimulate the expert.

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