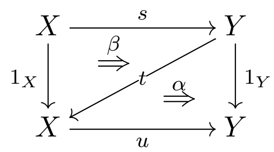
CONSEQUENCES OF SPLITTING IDEMPOTENTS: ADDENDUM

ROSS STREET

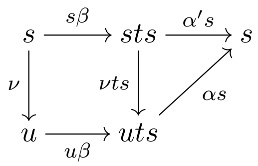
2010 Mathematics Subject Classification: 18D05; 18A40
Key words and phrases: idempotent; adjoint; dual.

Proposition 0.1. *In a bicategory $\mathscr{M}$, if the diagram*



pastes to give a retraction $\alpha s \centerdot u\beta \colon u \Longrightarrow s$ and idempotents split in $\mathscr{M}(Y,X)(t,t)$ then s has a right adjoint which is a retract of t.

Proof. Let ν be a right inverse for $\alpha s \centerdot u\beta$. Put $\alpha' = \alpha \centerdot \nu t \colon st \Longrightarrow 1_Y$. Then the lower path in the commutative diagram



is the identity. Then, by the Paré argument [1], a right adjoint to s is obtained by splitting the idempotent $t\alpha' \centerdot \beta t$. $\square$

References

[1] Saunders Mac Lane, *Categories for the Working Mathematician*, Graduate Texts in Mathematics **5** (Springer-Verlag, 1971).

Date: March 6, 2017.