THE CANTOR-LAWVERE DIAGONAL ARGUMENT

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Let $\mathscr{C}$ denote a category equipped with a functor $\otimes : \mathscr{C} \times \mathscr{C} \to \mathscr{C}$. A *comagma* in $\mathscr{C}$ is an object X equipped with a morphism $\delta_X : X \to X \otimes X$. A morphism $u : X \to Y$ is a *comagma morphism* when $\delta_X \circ u = (u \otimes u) \circ \delta_Y$.

Suppose A is a comagma. The *Lawvere property at an object* T for a morphism $g : A \otimes A \to Y$ asserts that, for all $f : A \to Y$, there exists a morphism $x : T \to A$ such that $g \circ \delta_A \circ x = f \circ x$.

Proposition 0.1. Suppose A is a comagma in the monoidal category $\mathscr{C}$ and $g : A \otimes A \to Y$ has the Lawvere property at T. Then, for all $t : Y \to Y$, there exists $y : T \to Y$ with $t \circ y = y$.

Proof. Apply the Lawvere property to $f = (A \xrightarrow{\delta_A} A \otimes A \xrightarrow{g} Y \xrightarrow{t} Y)$ to obtain a comagma morphism $x : T \to A$ such that $g \circ \delta_A \circ x = f \circ x$. Take $y = (T \xrightarrow{x} A \xrightarrow{\delta_A} A \otimes A \xrightarrow{g} Y)$ so that

$$y = g \circ \delta_A \circ x = f \circ x = t \circ g \circ \delta_A \circ x = t \circ y \ ,$$

as required. □

References

[1] F. William Lawvere, Diagonal arguments and cartesian closed categories, *Reprints in Theory and Applications of Categories* **15** (2006) 1–13; originally published as: *Lecture Notes in Mathematics* **92** (Springer, Berlin, 1969) 134–145.

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