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FUNCTORIAL KNOT THEORY: CATEGORIES OF TANGLES, COHERENCE, CATEGORICAL DEFORMATIONS, AND TOPOLOGICAL INVARIANTS
(Series on Knots and Everything 26)

By David N. Yetter: 230 pp., £40.00, isbn 981-02-4443-6
(World Scientific, Singapore, 2001).

Bull. London Math. Soc. 35 (2003)
DOI: 10.1112/S0024609302221864

By now, low-dimensional topologists claim David Yetter as their own. Yet he remains committed to his roots as an excellent category theorist. The valuable contributions the author has made to both these fields establish his credentials to create this unique book.

Knot theory is a wonderful subject that, for a century, existed quite happily with the understanding that the closest it need come to category theory was through their common ancestor, algebraic topology. Of course, it is still possible to make deep contributions to knot theory without touching categories. However, this book deals with the exciting contacts established since the mid-nineteen-eighties.

Likewise, the use of string-like diagrams to inspire and perform calculations is not new. Some examples are electrical circuit diagrams, Feynman diagrams in physics, the Penrose tensor calculus, the Brauer algebra in group representation theory, the Eilenberg–Kelly naturality calculus, flow charts in computing, and proof nets in logic. What has come forth in the last two decades is the formalisation of these *ad hoc* examples, to the mutual benefit of string theory and algebra.

The idea that there is algebraic structure married to geometry goes back at least to Felix Klein, who distinguished geometries in terms of their groups of transformations. The use of groups in geometry has been an incredible success story. In knot theory, Artin's braid groups play an important role. So now, in retrospect, I see the step from groups to monoidal categories as a natural extension of this history. 1985 was an interesting year. I was privileged to begin collaboration with André Joyal on braided monoidal categories: Artin's braid groups assembled themselves into the initial (or free) such structure. The same year, the Jones polynomial appeared in [**5**]. Quickly following up Jones' paper was evidence in [**4**] of Yetter's impressive entry into knot theory. Independently of my work with Joyal, Yetter was developing his monoidal category of tangles [**7**] to accommodate both links and braids. On learning of our work, Peter Freyd and Yetter recognised the existence of a braiding on Yetter's tangle category; but what is more, they realised that all objects have duals! Meanwhile the quantum group revolution was developing in Russia, as announced by V. G. Drinfeld at the 1986 Congress. Hopf algebras and quasi-triangular elements were fundamental to our circle of ideas. In particular, V. G. Turaev recognised Yang–Baxter operators in the monoidal category of tangles.

Thus began the subject of this book: the role of categorical structures in low-dimensional topology. What is needed is the part of category theory dealing with structural embellishments. There are no pullbacks or coequalizers to be found! What we see are monoidal categories, whose morphisms are geometric structures, and which are free in an appropriate sense. The same embellishments can be found on categories whose objects are algebraic structures. Therefore, using the freeness, functors from the geometric categories to the algebraic can be constructed. This procedure is a source of invariants for low-dimensional manifolds.

The self-contained first half of the book essentially culminates in the freeness property of the category of tangles on ribbons. It is pleasing to see the central role played by this theorem due to one of Macquarie University's star students, Mei Chee Shum [**6**]. Unfortunately, mathematics has lost her to the actuarial world.

The second half of the book concerns the infinitesimal deformation theory of braided monoidal categories, and a consequential approach to the theory of Vassiliev invariants. The starting point is a cochain complex associated with a monoidal functor, studied independently by Alexei Davydov [**2**], and by Louis Crane and the author [**1**]. This is the Hochschild complex of an algebra in a suitable linear convolution monoidal functor category, and so relates to classical Gerstenhaber deformation theory of an algebra in the category of vector spaces.

Perhaps the most serious typographical error is in the Definition 3.6 of bialgebra (p. 43), where the diagonal (or comultiplication) arrow goes in the wrong direction. Definition 3.29 of monoidal equivalence (p. 57) is technically correct, but it would be nice to point out that the concept behaves algebraically: it amounts to a (strong) monoidal functor which is an equivalence as a mere functor. I would not have associated the usual tensor product of enriched categories (Definition 10.10) with Deligne; in Section 5 of [**3**], there is a tensor product of abelian categories for which that would be more apt. A final minor complaint is that the author becomes somewhat bogged down in idiosyncratic technicalities on monoidal categories around Lemmas 3.26 and 3.27. Otherwise, the book is very economically written without being terse.

While the book, by its own admission, is not the last word on the subject, it is a timely and readable piece, and will expose the delights of two interconnected mathematical disciplines to a wide audience of graduate students and professional mathematicians.

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INVARIANT THEORY OF FINITE GROUPS
(Mathematical Surveys and Monographs 94)

By MARA D. NEUSEL and LARRY SMITH: 371 pp., US$81.00, ISBN 0-8218-2916-5 (American Mathematical Society, Providence, RI, 2002).

Bull. London Math. Soc. 35 (2003)
DOI: 10.1112/S0024609302231860

Suppose that V is a finite-dimensional representation of a finite group G over a field F. Form the symmetric algebra on the dual space V^* of V, denoted by $F[V]$ and called the *algebra of polynomial functions on* V. The action of G on V induces an action on $F[V]$, and invariant theory is the study of the subalgebra of G-fixed elements $F[V]^G$. It is customary to consider $F[V]$ as a graded algebra, with the grading determined by declaring that every element of V^* has degree one. The action of G preserves the grading so that $F[V]^G$ is a graded subalgebra, a fact that is obvious mathematically, and which is very useful.

Part of the attraction of invariant theory is the fact that many difficult mathematical problems immediately arise from this seemingly simple set-up. In fact, one soon develops a healthy respect for the subject upon being introduced, in the opening chapter, to the seminal contributions of David Hilbert and Emmy Noether to the subject. For example, Noether showed that $F[V]^G$ is a finitely generated algebra, and Hilbert went further to show that each syzygy module of $F[V]^G$ is finitely generated and that the chain of syzygies is finite. Here, the first syzygy is the kernel of a surjection from a finite polynomial algebra onto $F[V]^G$, which exists by Noether's theorem, and the kth syzygy is constructed by replacing $F[V]^G$ by the $(k-1)$th syzygy. Hilbert's syzygy theorem is arguably the start of the subject known as homological algebra.

Hilbert and Noether left a number of interesting problems, which are given a thorough treatment in this volume. For example, what is the formula for the maximal degree of an element in a minimal generating set for $F[V]^G$? In particular, Noether conjectured that this maximal degree would be less than the order of G, provided that the order of G is invertible in F. This conjecture was recently proved, independently, by P. Fleischmann and J. Fogarty. This fact illustrates the recent flurry of research activity in invariant theory, with the majority of the four hundred and sixty-six bibliographical references in the book being post-1980. In particular, the mid-1970s saw the introduction into invariant theory of techniques from algebraic topology, such as the Poincaré duality and module structures over the Steenrod algebra and the remarkable T-functor of Jean Lannes, which had a profound effect. For example, I believe that the longstanding depth conjecture,