Schäppi, Daniel

The formal theory of Tannaka duality. (English) Zbl 1314.18008

Astérisque 357. Paris: Société Mathématique de France (SMF) (ISBN 978-2-85629-773-5/pbk). vii, 140 p. (2013).

The appeal of this memoir is that the author develops an attractive general theory, with its own significant theorems, then he returns to the motivating problem to deduce an elegant solution.

The initial motivation was a question of Richard Pink in the area of p-adic Galois representation theory. Something analogous to Tannaka duality seemed to be present with the categories of filtered modules involved, yet none of the many various existing approaches to Tannaka duality would fit.

In the reviewer's book [Quantum groups. A path to current algebra. Cambridge: Cambridge University Press (2007; Zbl 1117.16031)], the discussion of Tannaka duality based on an adjunction is analogous to the discussion of "structure and semantics" in his paper [J. Pure Appl. Algebra 2, 149–168 (1972; Zbl 0241.18003)]. The adjunction involved in the latter case arises from making explicit the universal property of the Eilenberg-Moore object for a monad, or equally, a comonad. The genius of the present memoir is in discovering the correct universal property for a Tannaka object for a comonad. It is a subtle variant of the defining property of Eilenberg-Moore object. A little weaker condition on 2-cells would work for the Tannaka adjunction of this memoir but the full force will be used in later work to lift autonomous structures involving extraordinary natural transformations. The stronger property is however shown later in the memoir to hold in examples. If the hom categories of the bicategory had equalizers preserved by composition, every Eilenberg-Moore object would be a Tannaka object: but this is not the case in the important example of $\mathbf{Mod}(\mathcal{V})$ examined in Chapter 5. The subtlety mentioned above captures, in these examples, the requisite finitary nature of the representations involved: this is where "Cauchy completeness" comes in. For modules, Cauchy completeness is about finite generation and projectivity.

As foreshadowed, the memoir proceeds to obtain reconstruction and recognition theorems. The recognition theorem simplifies in the case where the base $\mathcal{V}$ is abelian and the enriched categories are "tame". This allows the author to include recent results of *K. Szlachányi* ["Fiber functors, monoidal sites and Tannaka duality for bialgebroids", Preprint, `arXiv:0907.1578`].

There is a review of material from various sources on monoidal categories. However, the presentation here is easy to read and contains original ideas as well as nice organisation.

Chapter 12 on base change shows how the general set up leads to an improved understanding of Tannaka theory. Questions can now be asked in this context with applicable answers.

Finally, the four Appendices will be an important source for future developments.

Reviewer: Ross H. Street (North Ryde)

MSC:

18D10	Monoidal, symmetric monoidal and braided categories (MSC2010)
18D35	Structured objects in a category (MSC2010)
16T05	Hopf algebras and their applications
16T10	Bialgebras
16T15	Coalgebras and comodules; corings

Keywords:

Tannakian category; Galois representation theory; Eilenberg-Moore object; monoidal category

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